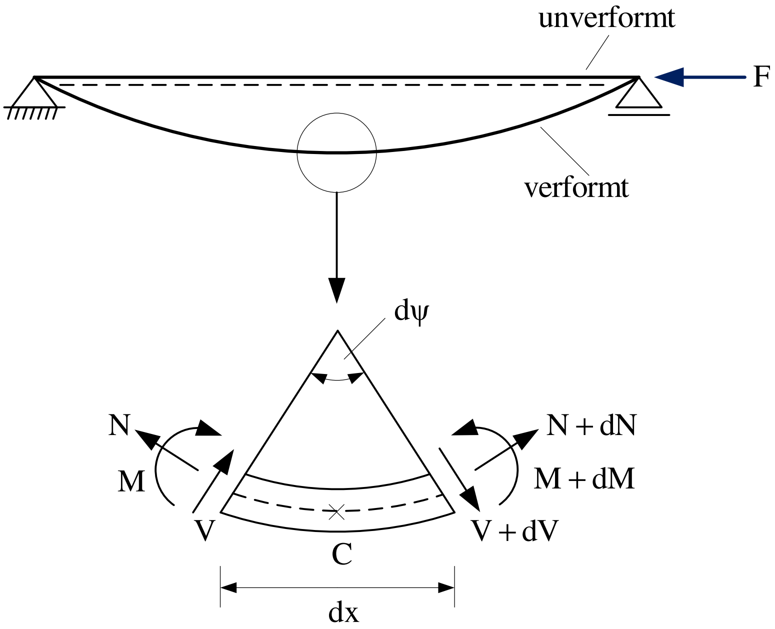
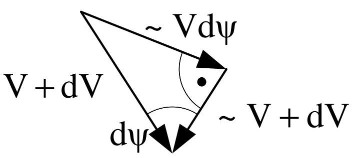
Herleitung der Knickgleichung

Gleichgewicht am verformten System:



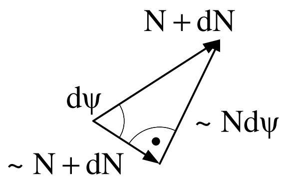
 :

$$dN + Vd\psi = 0 \qquad (1)$$



 :

$$dV - Nd\psi = 0 \qquad (2)$$



 c :

$$dM - Vdx = 0 \qquad (3)$$

(3) in (1):

$$\frac{dN}{dx} = -V\frac{d\psi}{dx} = -\frac{dM}{dx}\cdot\frac{d\psi}{dx} = \underbrace{-\frac{d}{dx}\left(EI\frac{d\psi}{dx}\right)\frac{d\psi}{dx}}_{\text{klein von höherer Ordnung!}} \approx 0$$

$\rightarrow \quad N = \text{const.} = -F \qquad$ (Äußere Drucklast wird von N übertragen)

Aus (2):

$$\frac{dV}{dx} + F \cdot \frac{d\psi}{dx} = 0$$

$$V = \frac{dM}{dx} \quad , \quad M = EI \frac{d\psi}{dx} \quad , \quad \psi = -w'$$

$$\rightarrow \left(EI \cdot w''\right)'' + F \cdot w'' = 0$$

Bei EI = const.: $EI \cdot w^{IV} + F \cdot w'' = 0$

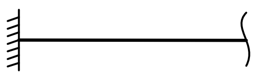
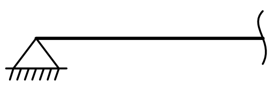
Knickgleichung: $\boxed{w^{IV} + \lambda^2 w'' = 0}$ mit: $\lambda^2 = \frac{F}{EI}$

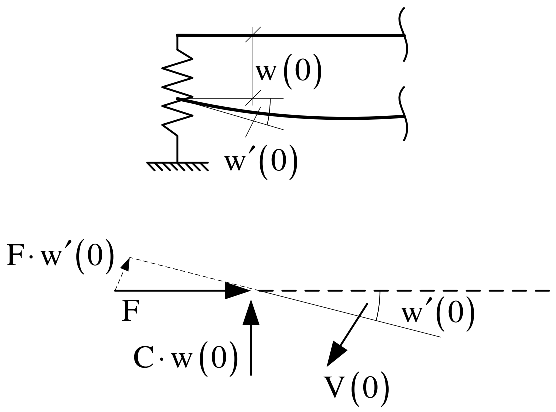
<u>Allgemeine Lösung der Differentialgleichung 4.Ordnung:</u>

$$\boxed{w(x) = A \cdot \cos(\lambda x) + B \cdot \sin(\lambda x) + C \cdot \lambda \cdot x + D}$$

A, B, C, D - unbekannte Konstanten, die aus den Randbedingungen bestimmt werden

Randbedingungen

	$w(0) = 0 \;,\; w'(0) = 0$
	$w(0) = 0 \;,\; M(0) = 0 \;\; \left(M(0) = -EI \cdot w''(0)!\right)$

<table>
<tr><td>


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$M(0)=0 \Rightarrow EI\cdot w''(0)=0$

$V(0)=c\cdot w(0)+F\cdot w'(0)$

$\Rightarrow \quad EI\cdot w'''(0)+c\cdot w(0)+F\cdot w'(0)=0$

$\left(V(0)=-EI\cdot w'''(0)\ !\right)$

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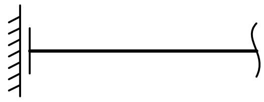
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$w'(0)=0\ ,\ V(0)=0\ \left(EI\cdot w'''(0)=0\right)$

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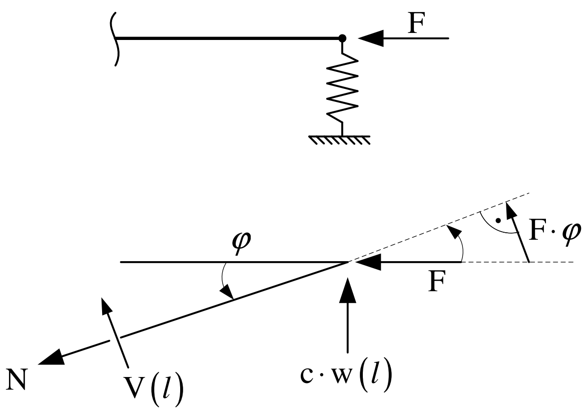

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$V(l)=-c\cdot w(l)-F\cdot\varphi=-c\cdot w(l)+F\cdot w'(l)$

$M(l)=0$

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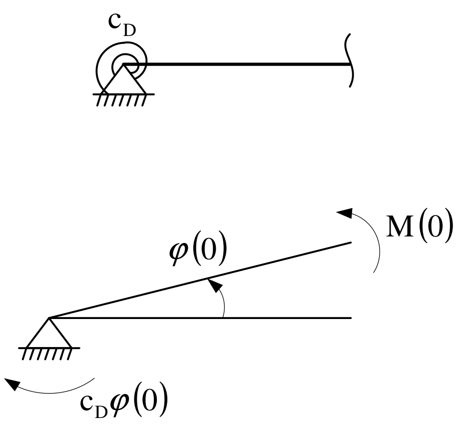

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$M(0)=c_D\cdot\varphi=-c_D\cdot w'(0)$

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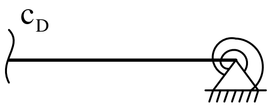
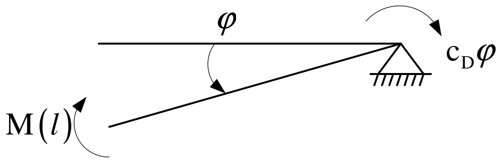
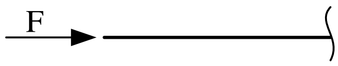
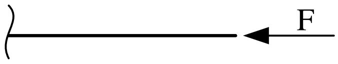
 	$M(l) = -c_D \cdot \varphi = c_D \cdot w'(l)$
	$V(0) = F \cdot w'(0)$
	$V(l) = F \cdot w'(l)$