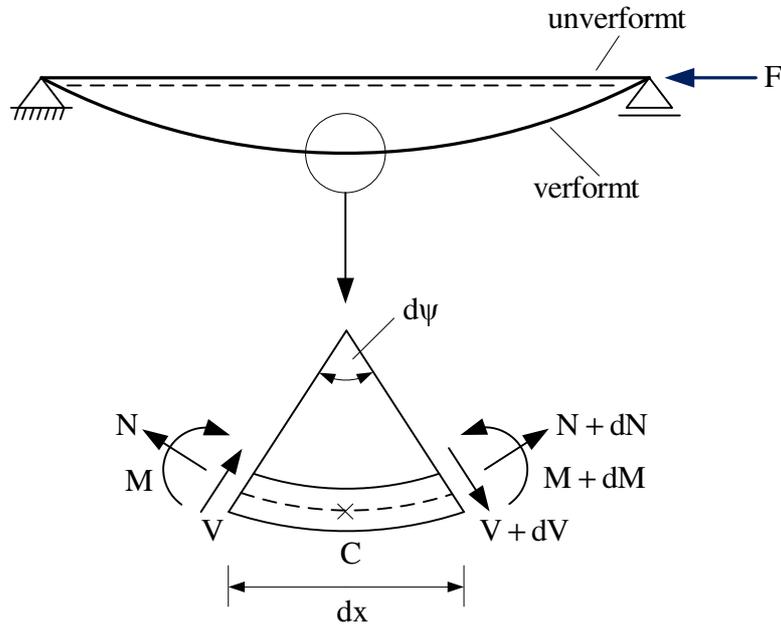


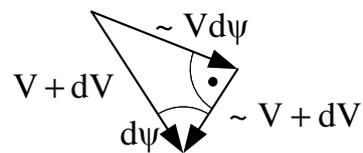
Herleitung der Knickgleichung

Gleichgewicht am verformten System:



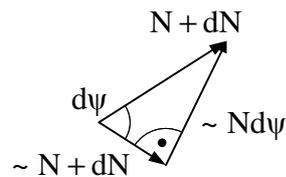
↖ :

$$dN + Vd\psi = 0 \quad (1)$$



↗ :

$$dV - Nd\psi = 0 \quad (2)$$



↻ c:

$$dM - Vdx = 0 \quad (3)$$

(3) in (1):

$$\frac{dN}{dx} = -V \frac{d\psi}{dx} = -\frac{dM}{dx} \cdot \frac{d\psi}{dx} = -\underbrace{\frac{d}{dx} \left(EI \frac{d\psi}{dx} \right)}_{\text{klein von höherer Ordnung!}} \frac{d\psi}{dx} \approx 0$$

klein von höherer Ordnung!

$$\rightarrow N = \text{const.} = -F \quad (\text{Äußere Drucklast wird von } N \text{ übertragen})$$

Aus (2):

$$\frac{dV}{dx} + F \cdot \frac{d\psi}{dx} = 0$$

$$V = \frac{dM}{dx}, \quad M = EI \frac{d\psi}{dx}, \quad \psi = -w'$$

$$\rightarrow (EI \cdot w'')'' + F \cdot w'' = 0$$

Bei $EI = \text{const.}$: $EI \cdot w^{IV} + F \cdot w'' = 0$

Knickgleichung:

$$w^{IV} + \lambda^2 w'' = 0$$

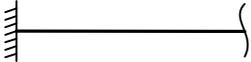
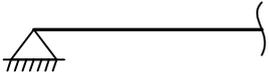
mit: $\lambda^2 = \frac{F}{EI}$

Allgemeine Lösung der Differentialgleichung 4.Ordnung:

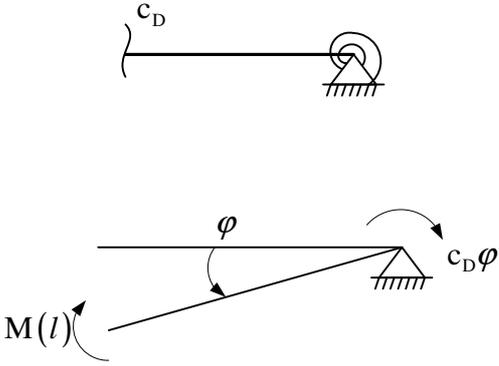
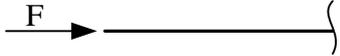
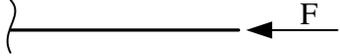
$$w(x) = A \cdot \cos(\lambda x) + B \cdot \sin(\lambda x) + C \cdot \lambda \cdot x + D$$

A, B, C, D - unbekannte Konstanten, die aus den Randbedingungen bestimmt werden

Randbedingungen

	$w(0) = 0, \quad w'(0) = 0$
	$w(0) = 0, \quad M(0) = 0 \quad (M(0) = -EI \cdot w''(0)!) $

	$M(0) = 0 \Rightarrow EI \cdot w''(0) = 0$ $V(0) = c \cdot w(0) + F \cdot w'(0)$ $\Rightarrow EI \cdot w'''(0) + c \cdot w(0) + F \cdot w'(0) = 0$ $(V(0) = -EI \cdot w'''(0) !)$
	$w'(0) = 0, \quad V(0) = 0 \quad (EI \cdot w'''(0) = 0)$
	$V(l) = -c \cdot w(l) - F \cdot \varphi = -c \cdot w(l) + F \cdot w'(l)$ $M(l) = 0$
	$M(0) = c_D \cdot \varphi = -c_D \cdot w'(0)$

 <p>The diagram shows a horizontal beam of length l fixed at the right end to a rotational spring with constant c_D. The rotation at the right end is denoted by φ. The bending moment at the left end is denoted by $M(l)$.</p>	$M(l) = -c_D \cdot \varphi = c_D \cdot w'(l)$
 <p>The diagram shows a horizontal beam with a force F applied at the left end, pointing to the right.</p>	$V(0) = F \cdot w'(0)$
 <p>The diagram shows a horizontal beam with a force F applied at the right end, pointing to the left.</p>	$V(l) = F \cdot w'(l)$