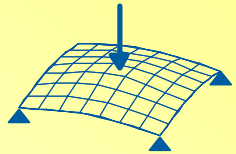
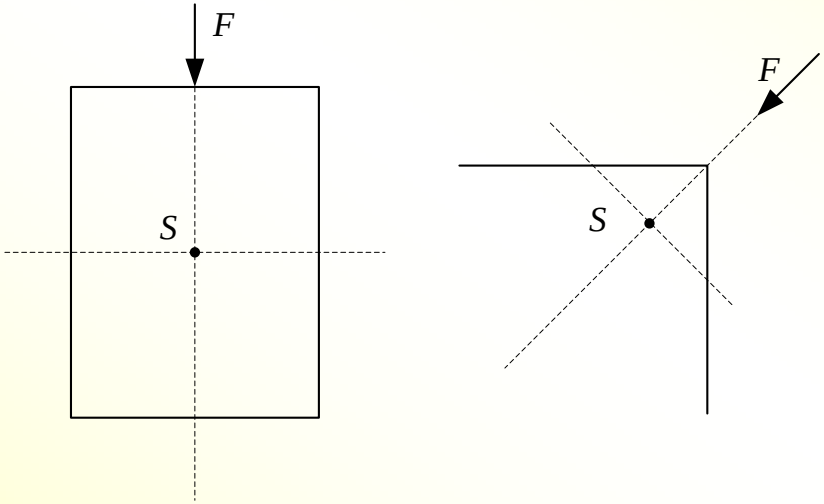
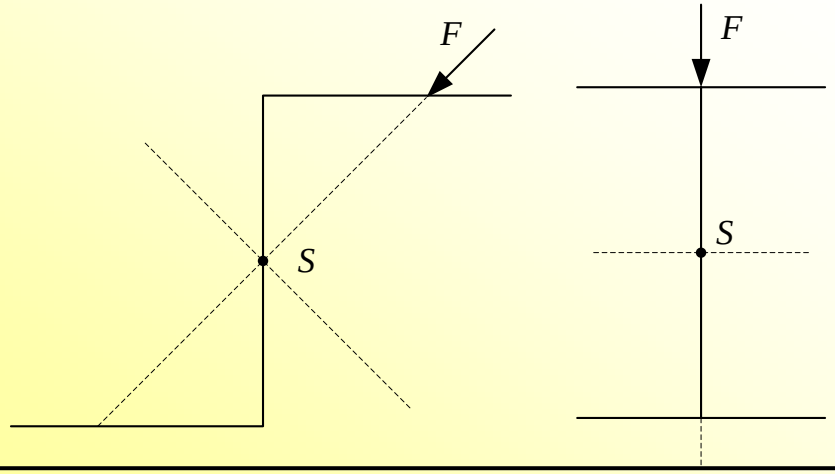
3.6 Schiefe Biegung

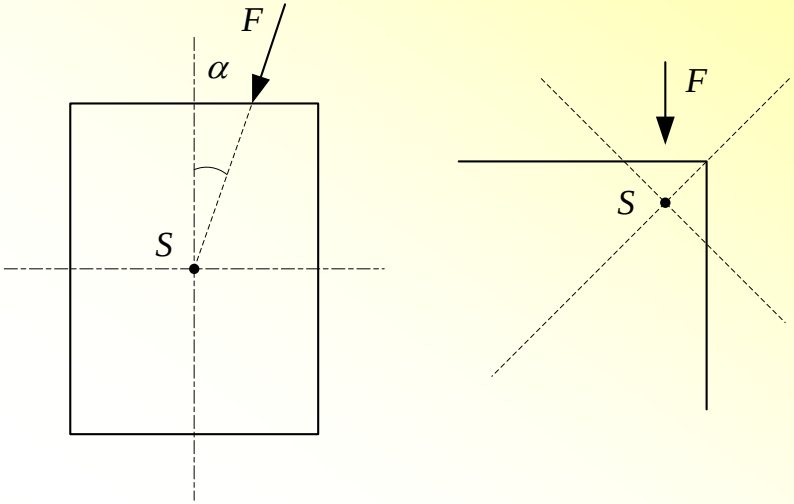


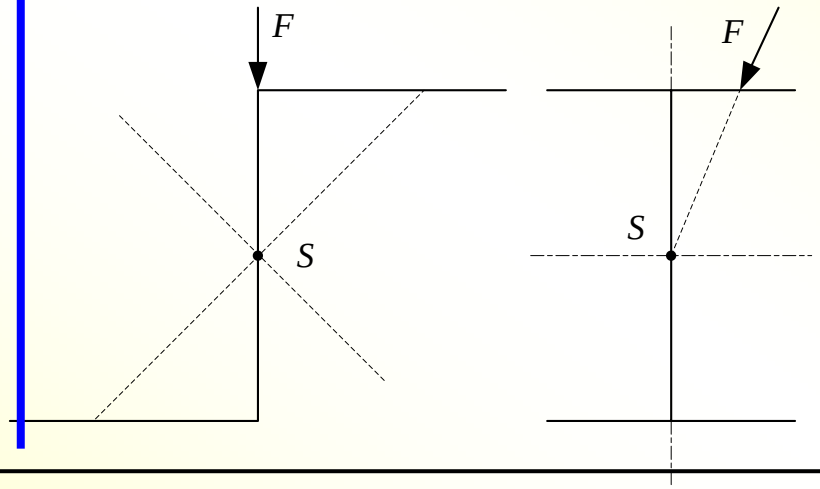
Gerade Biegung



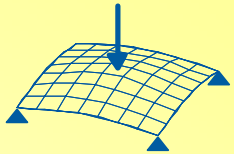


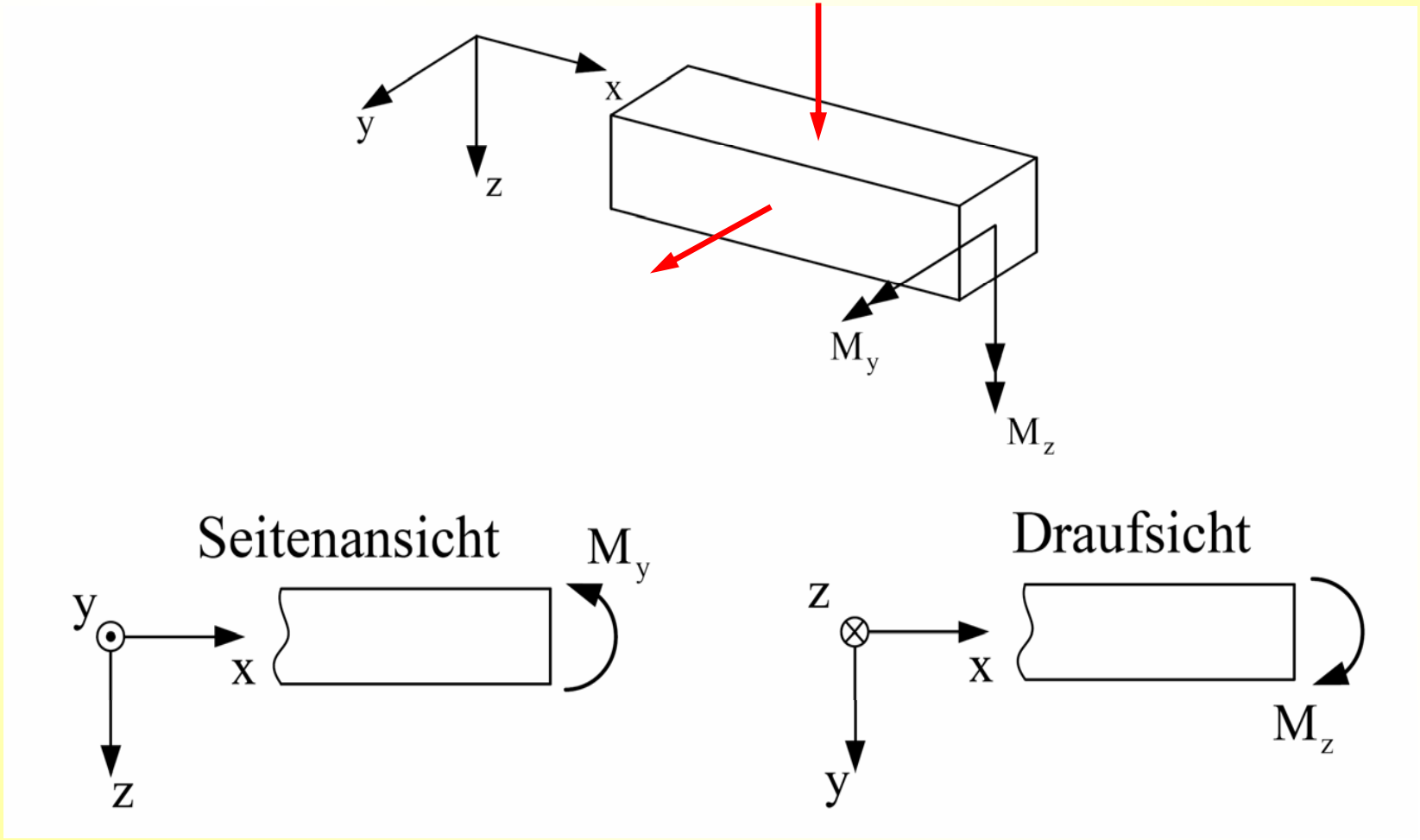
Schiefe Biegung





3.6 Schiefe Biegung

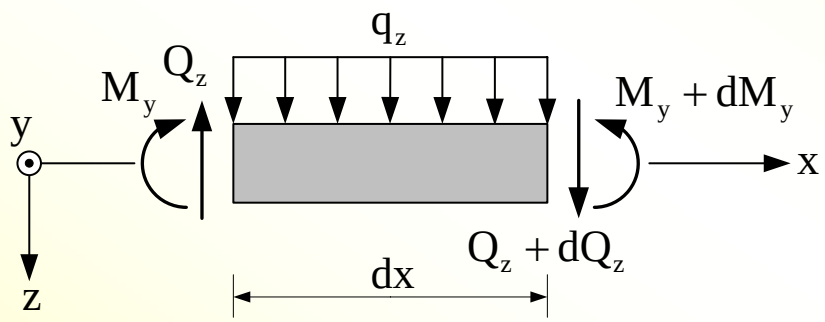




3.6 Schiefe Biegung

1.) Gleichgewichtsgleichungen

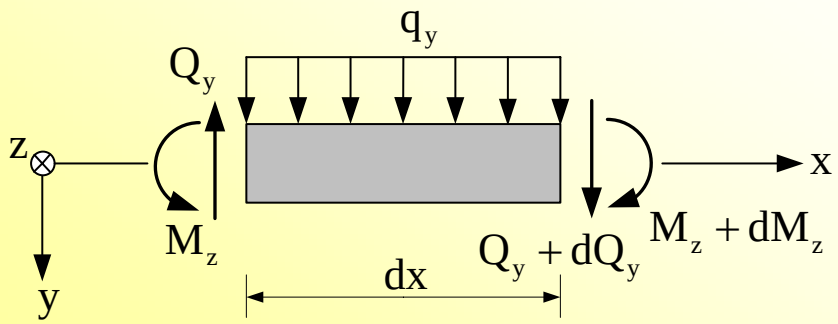
Seitenansicht



$$\frac{dM_y}{dx} = V_z$$

$$\frac{dV_z}{dx} = -p_z$$

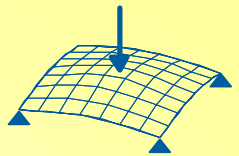
Draufsicht



$$\frac{dM_z}{dx} = -V_y$$

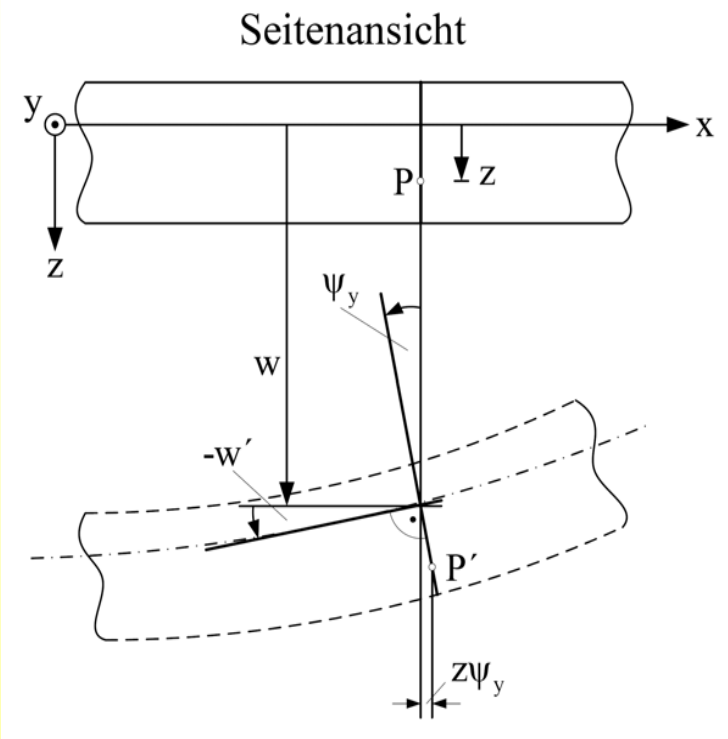
$$\frac{dV_y}{dx} = -p_y$$

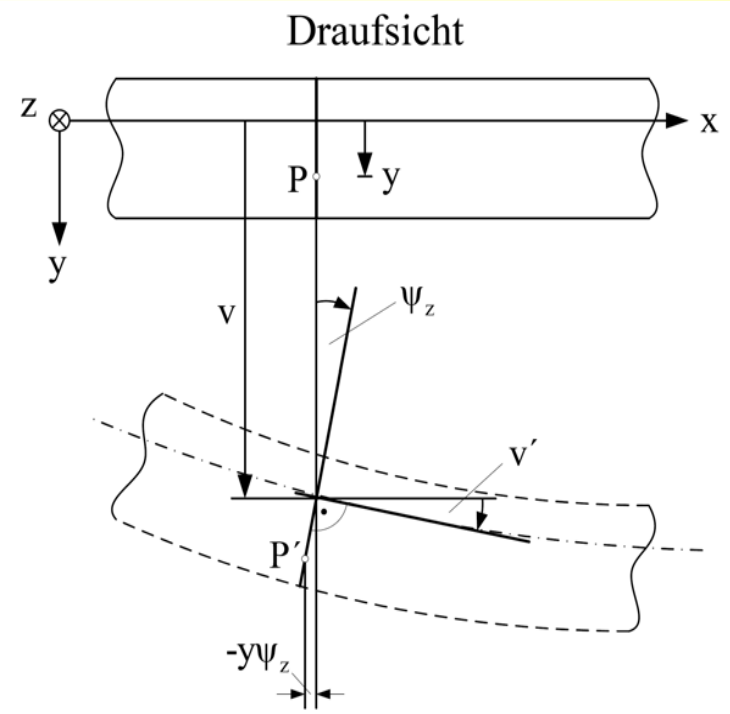
3.6 Schiefe Biegung



2.) Elastizitätsgesetz $\sigma = E \cdot \varepsilon$

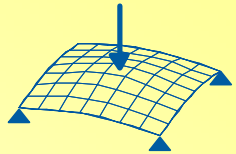
3.) Kinematik





- **Verschiebung in x-Richtung** $u = \psi_y(x) \cdot z - \psi_z(x) \cdot y$
- **Drehwinkel** $\psi_y(x) = -w'(x); \qquad \psi_z(x) = v'(x)$

3.6 Schiefe Biegung



- **Dehnung in x-Richtung**

$$\varepsilon = \frac{du}{dx} = \psi_y'(x) \cdot z - \psi_z'(x) \cdot y = -w''(x) \cdot z - v''(x) \cdot y$$

4.) Schnittgrößen und Spannungen

- **Normalkraft**

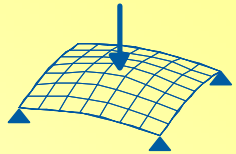
$$N = \int_A \sigma dA = \int_A E \cdot \varepsilon dA = -Ew'' \int_A z dA - Ev'' \int_A y dA = -Ew'' S_y - -Ev'' S_z = 0$$

(S_y=0 und S_z=0 da y-z-Achsen Schwerachsen sind!)

- **Biegemoment**

$$M_y = \int_A z \cdot \sigma dA = \int_A z \cdot E\varepsilon dA = -Ew'' \int_A z^2 dA - Ev'' \int_A yz dA = -EI_y w'' + EI_{yz} v''$$

3.6 Schiefe Biegung



$$M_z = -\int_A y \cdot \sigma dA = -\int_A y \cdot E\varepsilon dA = Ew''\int_A yzdA + Ev''\int_A y^2 dA = -EI_{yz}w'' + EI_z v''$$

5.) Differentialgleichungen für die Biegelinien

Möglichkeit I:

$$-EI_y w'' + EI_{yz} v'' = M_y$$
$$-EI_{yz} w'' + EI_z v'' = M_z$$

$$Ew'' = \frac{1}{\Delta}\left(-M_y I_z + M_z I_{yz}\right)$$
$$Ev'' = \frac{1}{\Delta}\left(-M_y I_{yz} + M_z I_y\right)$$

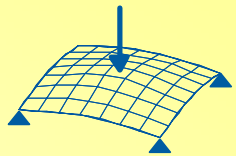
$$\Delta = I_y I_z - I_{yz}^2$$

Entkoppelte Dgln 2. Ordnung!

Sonderfall: Im Hauptachsensystem $I_{yz} = 0$

$$EI_y w'' = -M_y$$
$$EI_z v'' = M_z$$

3.6 Schiefe Biegung



Möglichkeit II:

$$M_y = -EI_y w'' + EI_{yz} v''$$

$$V_z = M_y' = \left(-EI_y w'' + EI_{yz} v''\right)'$$

$$V_z' = \left(-EI_y w'' + EI_{yz} v''\right)'' = -p_z$$

$$E\left(I_y w'' - I_{yz} v''\right)'' = p_z$$

$$M_z = -EI_{yz} w'' + EI_z v''$$

$$V_y = -M_z' = \left(EI_{yz} w'' - EI_z v''\right)'$$

$$V_y' = \left(EI_{yz} w'' - EI_z v''\right)'' = -p_y$$

$$E\left(-I_{yz} w'' + I_z v''\right)'' = p_y$$

Gekoppelte Dgln 4. Ordnung !

3.6 Schiefe Biegung

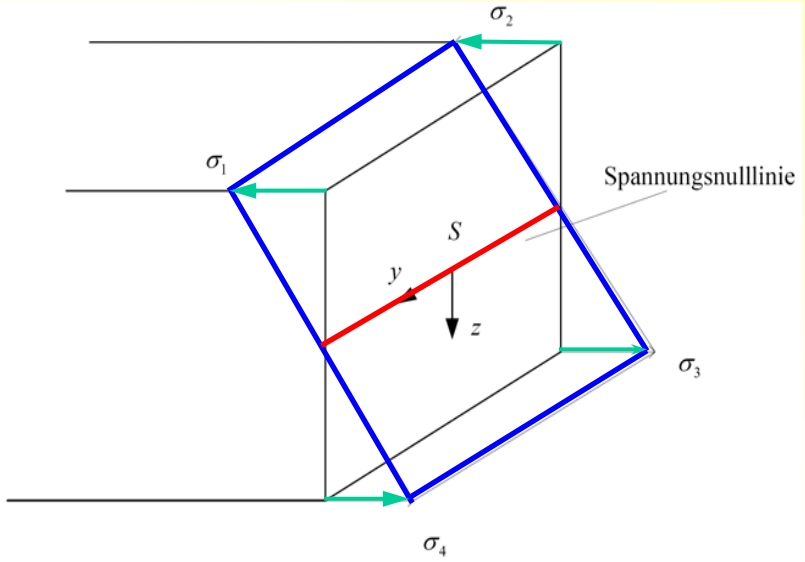
6.) Normalspannung

$$\sigma = E\varepsilon = -E\left(w''z - v''y\right) = \frac{1}{\Delta}\left[\left(M_y I_z - M_z I_{yz}\right)z - \left(M_z I_y - M_y I_{yz}\right)y\right]$$

σ ist linear über y und z verteilt!

Nulllinie im Querschnitt: $\sigma = 0$

$$\frac{z}{y} = \frac{M_z I_y - M_y I_{yz}}{M_y I_z - M_z I_{yz}}$$



Die größte Spannung σ_{max} tritt im Punkt mit dem größten Abstand von der Nulllinie auf!

Im Hauptachsensystem: $I_{yz} = 0$ ⟹

$$\frac{z}{y} = \frac{M_z I_y}{M_y I_z}$$

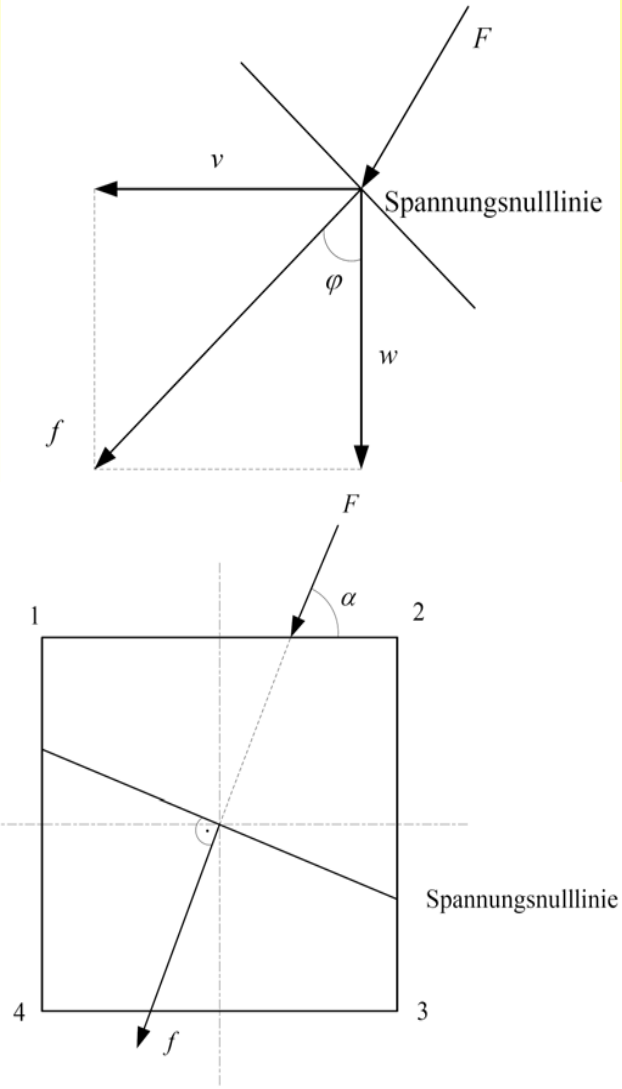
3.6 Schiefe Biegung

7.) Bestimmung der Durchbiegung f

- Bestimmung von w und v;
- Vektorielle Addition zur gesamten Durchbiegung f.

$$f = \sqrt{w^2 + v^2}$$

$$\tan\varphi = \frac{|v|}{|w|}$$



Bemerkungen:

- Die Durchbiegung f steht senkrecht zur Spannungs-nulllinie, i. A. aber nicht parallel zur Lastrichtung.
- Die Spannungsnulllinie steht i. A. nicht senkrecht zur Lastrichtung!