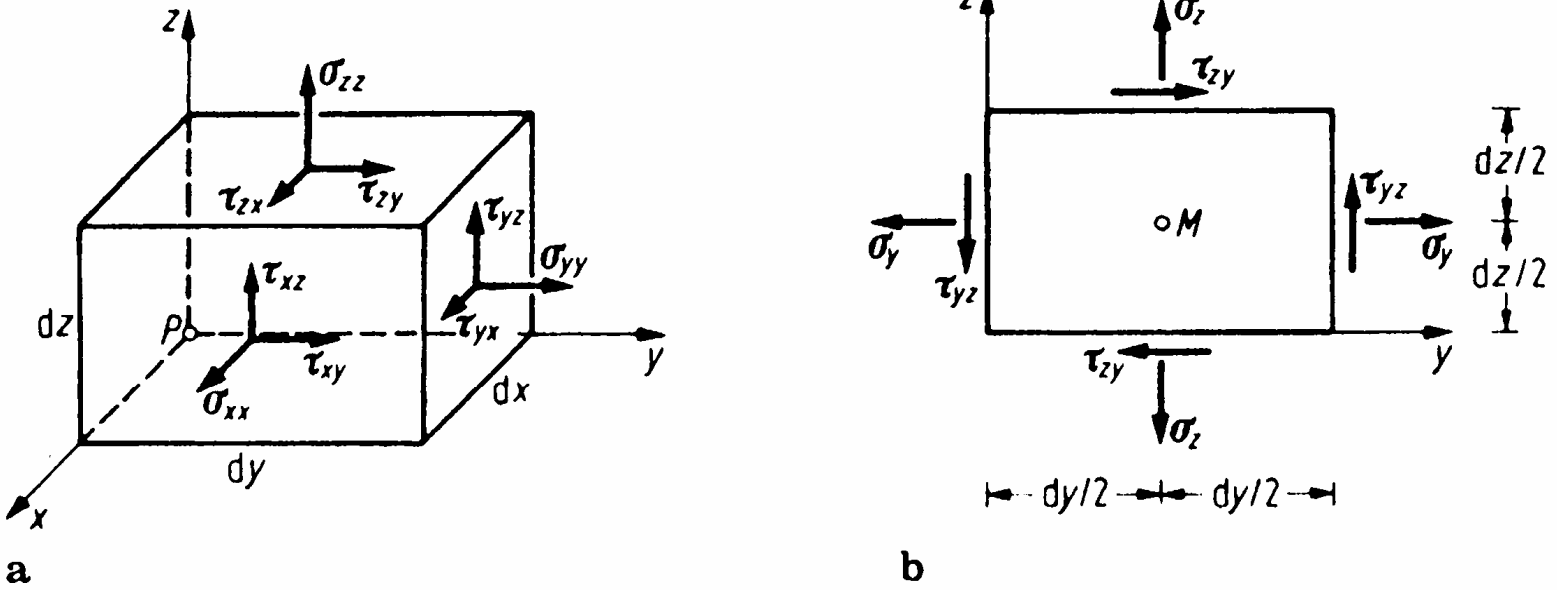
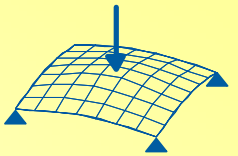
Spannungen



Positive Spannungen zeigen an einem positiven (negativen) Schnittufer in die positive (negative) Koordinatenrichtung!

Schubspannungen in 2 senkrecht aufeinander stehenden Schnitten (z.B. τ_{xy} und τ_{yx}) sind gleich!

Spannungstensor



Ebener Spannungszustand:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$

Räumlicher Spannungszustand:

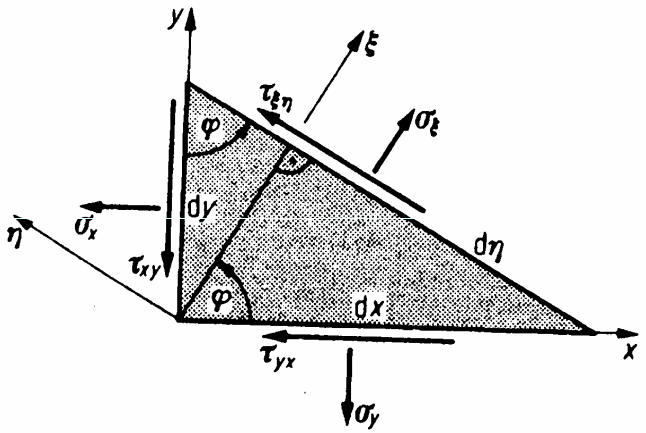
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Transformationsbeziehungen

$$\sigma_\xi = \frac{1}{2}\left(\sigma_x + \sigma_y\right) + \frac{1}{2}\left(\sigma_x - \sigma_y\right)\cos(2\varphi) + \tau_{xy}\sin(2\varphi)$$

$$\sigma_\eta = \frac{1}{2}\left(\sigma_x + \sigma_y\right) - \frac{1}{2}\left(\sigma_x - \sigma_y\right)\cos(2\varphi) - \tau_{xy}\sin(2\varphi)$$

$$\tau_{\xi\eta} = -\frac{1}{2}\left(\sigma_x - \sigma_y\right)\sin(2\varphi) + \tau_{xy}\cos(2\varphi)$$

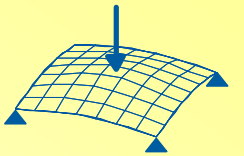


Spannungsinvarianten:

$$\sigma_\xi + \sigma_\eta = \sigma_x + \sigma_y$$

$$\mathrm{Det}\left(\boldsymbol{\sigma}\right) = \sigma_\xi \cdot \sigma_\eta - \tau_{\xi\eta}^2 = \sigma_x \cdot \sigma_y - \tau_{xy}^2$$

Hauptspannungen



Hauptspannungen:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan(2\varphi^*) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Hauptschubspannungen:

$$\tau_{\max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\frac{1}{2}(\sigma_1 - \sigma_2)$$

$$\tan(2\varphi^{**}) = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\sigma_M = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(\sigma_1 + \sigma_2)$$