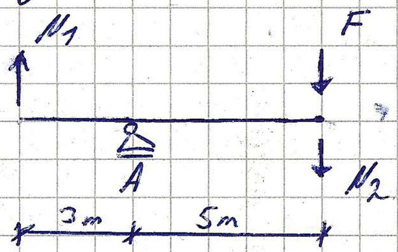
Aufgabe 1



$\overset{\curvearrowleft}{A}: \; N_1 \cdot 3 + N_2 \cdot 5 + F \cdot 5 = 0$

$\Rightarrow N_1 = -N_2 \cdot \frac{5}{3} - F \cdot \frac{5}{3}$

$\sigma_1 = \frac{N_1}{A} \Rightarrow N_1 = \sigma_1 \cdot A = 50 \cdot 100 = \underline{5000\ N}$

$\hookrightarrow N_2 = \underline{-13.000\ N}$

$\Rightarrow \sigma_2 = N_2/A_2 = \underline{-\,^{65}/_{6}\ N/mm^2}$

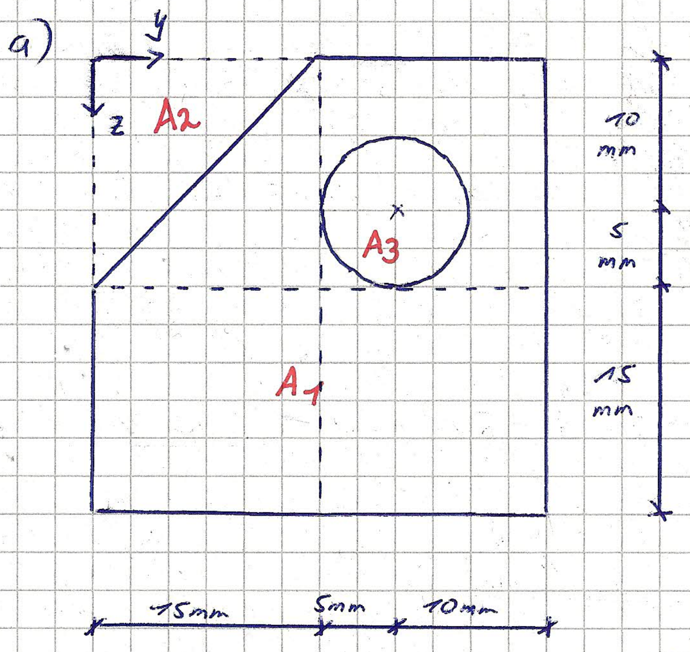
$\rightarrow \varepsilon = \frac{\sigma}{E} \Rightarrow \varepsilon_1 = \frac{\sigma_{max}}{E_1} = \frac{50}{30.000} = \underline{\frac{1}{600}}$

$\rightarrow \varepsilon_2 = \varepsilon_1 \cdot \frac{5}{3}$ (da $l_1 = l_2$) $\Rightarrow \underline{\varepsilon_2 = \frac{1}{360}}$

$\Rightarrow \Delta T = \frac{\varepsilon_2}{\alpha_T} = \frac{(1/360)}{5 \cdot 10^{-5}} = \underline{55{,}55\,°C}$

Aufgabe 2

a)



$A_1 = 30^2$; $y_1 = 15$; $z_1 = 15$

$A_2 = -\frac{15^2}{2}$; $y_2 = 5$; $z_1 = 5$

$A_3 = 5^2 \cdot \pi$; $y_3 = 20$; $z_3 = 10$

$$\bar{y}_s = \frac{\sum A_i \cdot y_i}{\sum A_i} = \underline{16{,}75\ mm} \; ; \; \bar{z}_s = \frac{\sum A_i \cdot z_i}{\sum A_i} = \underline{15{,}84\ mm^4}$$

	Eigenanteil → $I_{y,i}$	$I_{z,i}$	Steiner → I_y	I_z	I_{yz}	Eigenanteil I_{yz}
1	67500	67500	643	2764	-1.333	0
2	1406,3	1.406,3	-13239	-15538	14339	703,13
3	490,9	490,9	2683	828	1490	0

Summe: $\underline{I_y = 59.497{,}4\ mm^4}$; $\underline{I_z = 57.451{,}2\ mm^4}$

$\underline{I_{yz} = 15.199{,}13\ mm^4}$

b)

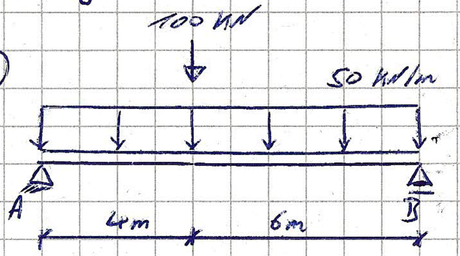
$$\boxed{\begin{aligned} I_{1,2} &= \frac{1}{2}(I_y + I_z) \pm \sqrt{\left(\frac{1}{2}\cdot(I_y - I_z)\right)^2 + I_{yz}^2} \\ \tan(2\varphi^*) &= 2 I_{yz} / (I_y - I_z) \end{aligned}}$$

$\rightarrow I_1 = \underline{73.705{,}3\ mm^4}$; $I_2 = \underline{43.237\ mm^4}$

$\varphi = \underline{43{,}08°}$

Aufgabe 3

a)

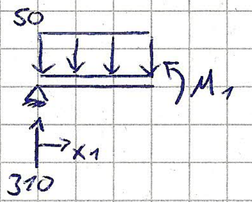


↑: $A_V + B_V - 100 - 50 \cdot 10 = 0$

$\overset{\frown}{A}$: $-B_V \cdot 10 + 100 \cdot 4 + 50 \cdot 10 \cdot 5 = 0$

↳ $\underline{A_V = 310 \text{ kN}}$; $\underline{B_V = 290 \text{ kN}}$

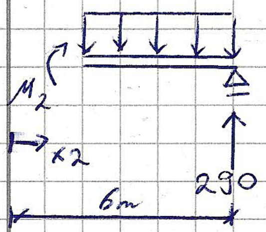
Momentenverlauf



$M_1 = 310 \cdot x_1 - 50 \cdot \frac{x_1^2}{2} = 310\, x_1 - 25\, x_1^2$

↳ $M_1(x_1 = 0) = \underline{0 \text{ kNm}}$

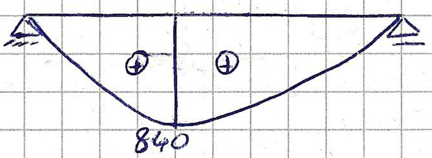
↳ $M_1(x_1 = 4) = \underline{840 \text{ kNm}}$



$\overset{\frown}{M_2} = 290 \cdot (6 - x_2) - 50 \cdot (6 - x_2) \cdot \frac{6 - x_2}{2}$

$= 840 + 10 \cdot x_2 - 25\, x_2^2$

↳ $M_2(x_2 = 0) = \underline{840 \text{ kNm}}$; $M_2(x_2 = 6) = \underline{0 \text{ kNm}}$



b)

Links:

$EI \cdot w_1'' = -M_1 = -310 \cdot x_1 + 25\, x_1^2$

$EI \cdot w_1' = -310 \cdot \frac{x_1^2}{2} + 25 \cdot \frac{x_1^3}{3} + C_1$

$EI \cdot w_1 = -310 \cdot \frac{x_1^3}{6} + 25 \cdot \frac{x_1^4}{12} + C_1 \cdot x_1 + C_2$

Rechts:

$EI \cdot w_2'' = -M_2 = -840 + 25\, x_2^2 - 10\, x$

$EI \cdot w_2' = -840 \cdot x_1 + \frac{25}{3} \cdot x_2^3 - 5\, x_2^2 + C_3$

$EI \cdot w_2 = -420 \cdot x_1^2 + \frac{25}{12} \cdot x_2^4 - \frac{5}{3}\, x_2^3 + C_3 \cdot x_2 + C_4$

Randbedingungen

Links : $w_1(0) = 0$; Rechts : $w_2(6) = 0$

Übergangsbedingung

$w_1(4) = w_2(0)$; $w_1'(4) = w_2'(0)$

Anwenden auf Gl. liefert:

$C_1 = 2723$; $C_2 = 0$; $C_3 = 776{,}\bar{6}$; $C_4 = 8120$

Einsetzen in w_1 und w_2 :

$$w_1 = \frac{x_1\left(6536 + x_1^2 \cdot (-124 + 5x_1)\right)}{96.000}$$

$$w_2 = \frac{(-6 + x_2) \cdot \left(-3248 + x_2\left(-852 + x_2 \cdot (26 + 5x_2)\right)\right)}{96.000}$$

c)

$w_1'(0) = 0{,}06808 \mathrel{\hat{=}} \underline{3{,}9^\circ}$

$w_2(4) = \left(w(8) =\right) \underline{0{,}123\ m}$

Aufgabe 4

$$\bar{I}_y = \frac{200^4}{12} - \frac{100^4}{12} = 1{,}25 \cdot 10^8 \ mm^4 = \bar{I}_z$$

$$\bar{I}_{yz} = 0$$

um 45° gedreht:

$$I_y = 1{,}25 \cdot 10^8 \ mm^4 = I_z \quad ; \quad I_{yz} = 0$$

$$M_y = 25.000.000 \ Nmm \quad ; \quad M_z = 0$$

$$\sigma = \frac{1}{\Delta} \cdot \left((M_y \cdot I_z - M_z \cdot I_{yz}) \cdot z - (M_z \cdot I_y - M_y \cdot I_{yz}) \cdot y \right)$$

$$= 0{,}2 \cdot z$$

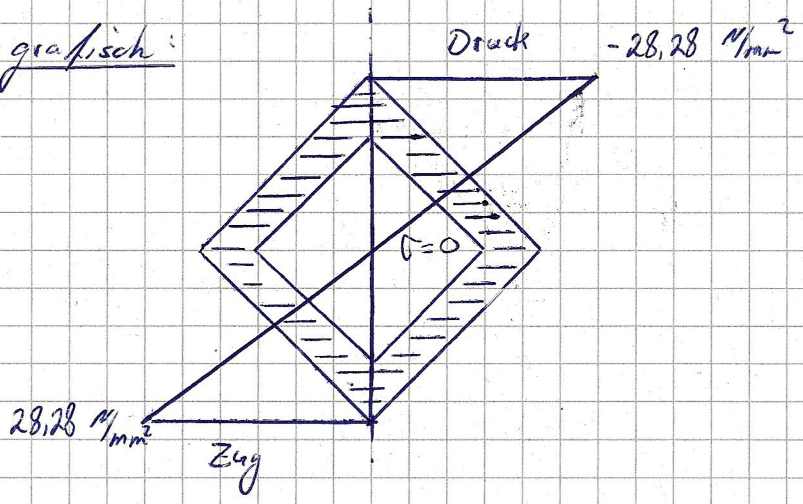
$\rightarrow 0 = 0{,}2 \cdot z \rightarrow z = 0$ Spannungsnulllinie

Eckpunkte:

$\sigma = 0{,}2 \cdot z$ $\qquad z = 141{,}4 \ mm \Rightarrow \sigma = 28{,}28 \ N/mm^2$

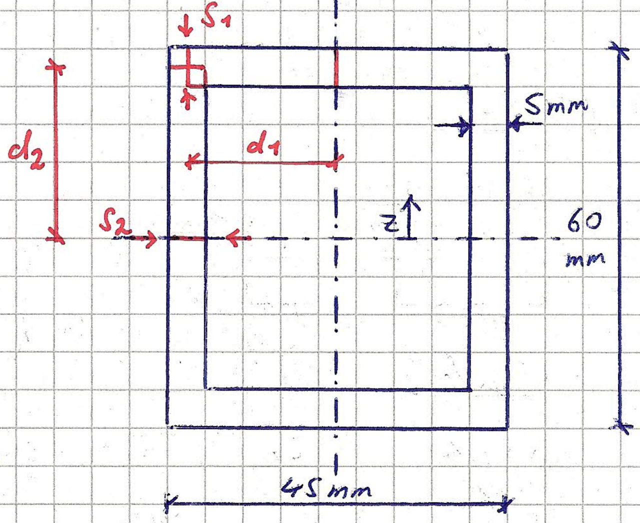
$\qquad z = -141{,}4 \ mm \Rightarrow \sigma = -28{,}28 \ N/mm^2$

grafisch:



Aufgabe 5

a) $V = 10\ kN$



Schnitte

$d_1 = \frac{45}{2} - \frac{5}{2} = 20\ mm$

$z_1 = \frac{60}{2} - \frac{5}{2} = 27{,}5\ mm$

$d_2 = \frac{60}{2} - \frac{5}{2} = 27{,}5\ mm$

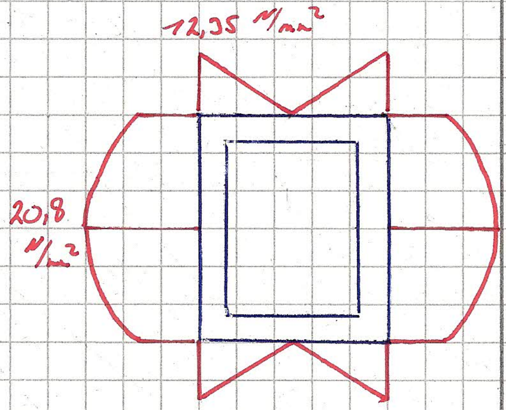
$z_2 = \frac{d_2}{2} = 13{,}75\ mm$

$\rightarrow S_y^{1-1} = d_1 \cdot 5 \cdot z_1 = \underline{2750\ mm^3}$

$\rightarrow S_y^{2-2} = S_y^{1-1} + d_2 \cdot 5 \cdot z_2 = \underline{4640{,}63\ mm^3}$

$I_y = \frac{60^3 \cdot 45}{12} - \frac{50^3 \cdot 35}{12} = 445.417\ mm^4$

Schubspannungsverlauf



$\tau_{1-1} = \frac{V \cdot S_y^{1-1}}{I_y \cdot t^{1-1}} = \frac{10.000 \cdot 2750}{445.417 \cdot 5}$

$= \underline{\underline{12{,}35\ N/mm^2}}$

$\tau_{2-2} = \frac{V \cdot S_y^{2-2}}{I_y \cdot t^{2-2}} = \frac{10.000 \cdot 4640{,}63}{445.417 \cdot 5}$

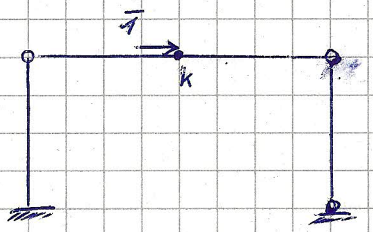
$= \underline{\underline{20{,}837\ N/mm^2}}$

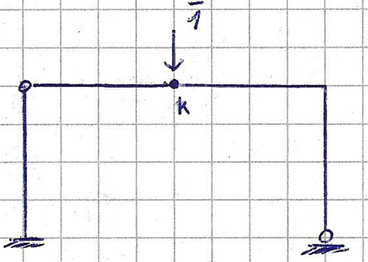
b) $\tau_{max} = \frac{M_T}{W_T}$; $M_T = 2\ kNm = 2 \cdot 10^3 \cdot 10^3\ N\,mm$

$W_T = 2 \cdot A_m \cdot t = 2 \cdot (40 \cdot 55) \cdot 5 = 22.000\ mm^3$

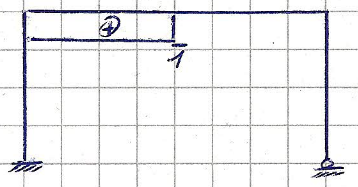
$\Rightarrow \underline{\underline{\tau_{max} = 90{,}9\ N/mm^2}}$

Aufgabe 6

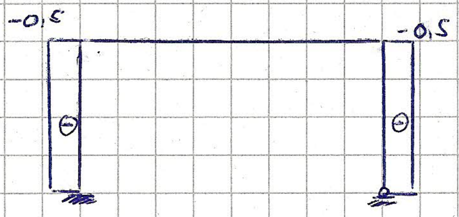




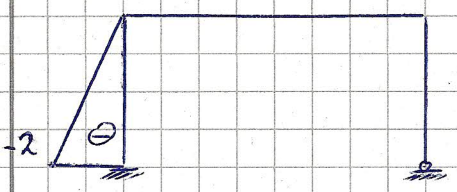
Normalkraftverlauf



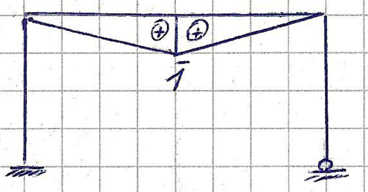
Normalkraftverlauf



Momentenverlauf



Momentenverlauf



Verschiebung horizontal:

$$\bar{1}\cdot u = \int \frac{N\cdot\bar{N}}{EA}\,dx + \int \frac{M\cdot\bar{M}}{EI}\cdot dx$$

$$u = \frac{1}{EA_R}\cdot 1\cdot 50\cdot 2 + \frac{1}{EI_S}\cdot(-2)\cdot(-100)\cdot\frac{1}{3}\cdot 2 = \underline{\underline{0{,}0068\ m}}$$

Verschiebung vertikal

$$\bar{1}\cdot w = \left(\frac{1}{EA_S}\cdot(-0{,}5)\cdot(-40)\cdot 2\right) + \left(\frac{1}{EA_S}\cdot(-0{,}5)\cdot(-90)\cdot 2\right)$$

$$+\left(\frac{1}{EI_R}\cdot\frac{5}{12}\cdot 40\cdot 1\cdot 2\right)\cdot 2$$

$$= \underline{\underline{0{,}0067\ m}}$$