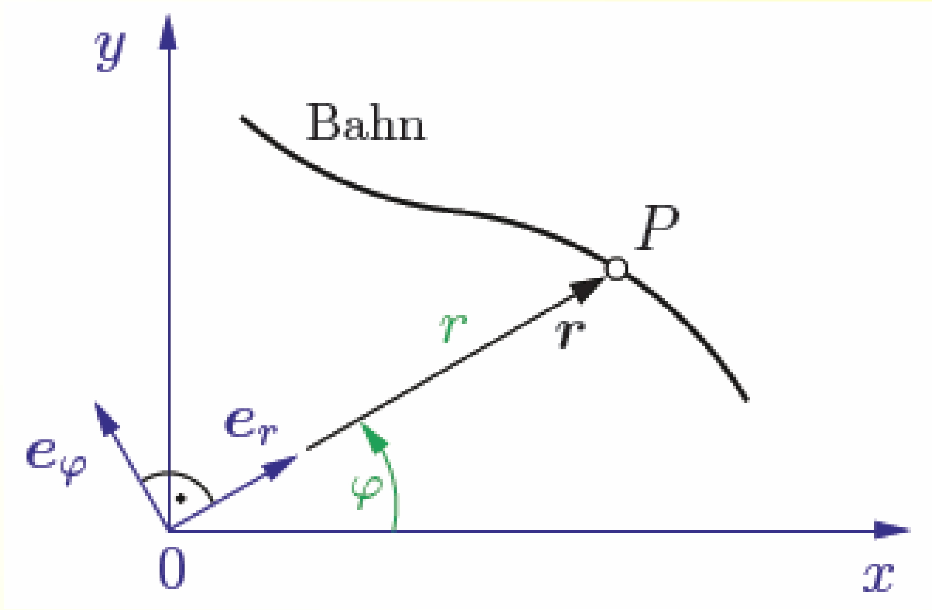
2.1.3 Ebene Bewegung in Polarkoordinaten

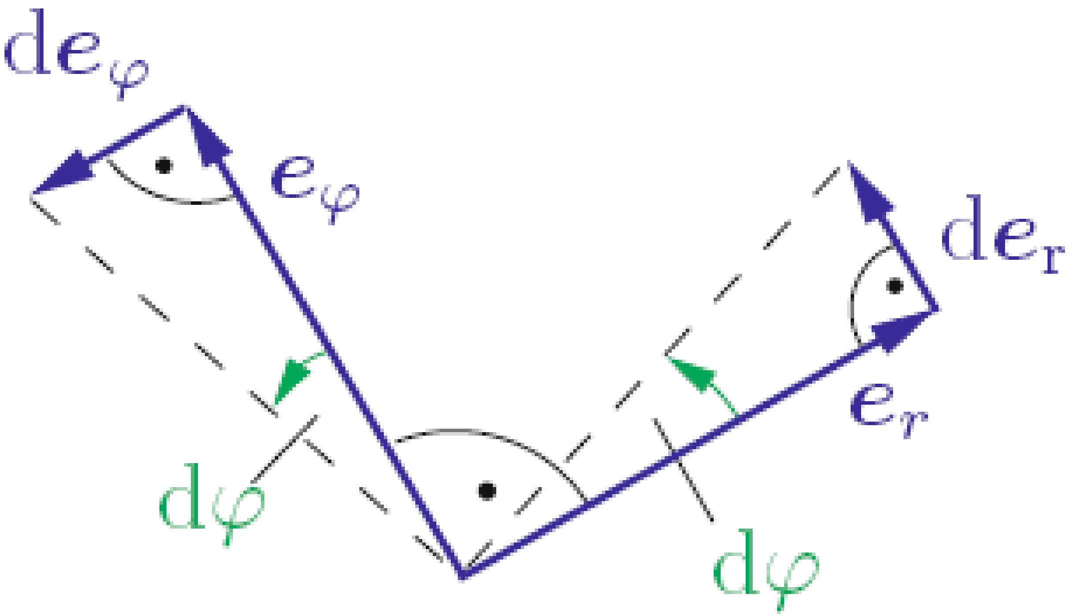


$$\boldsymbol{r} = r\,\boldsymbol{e}_r\,,$$

$$\boldsymbol{v} = v_r\,\boldsymbol{e}_r + v_\varphi\,\boldsymbol{e}_\varphi = \dot{r}\,\boldsymbol{e}_r + r\dot{\varphi}\,\boldsymbol{e}_\varphi,$$

$$\boldsymbol{a} = a_r\,\boldsymbol{e}_r + a_\varphi\,\boldsymbol{e}_\varphi = \left(\ddot{r} - r\dot{\varphi}^2\right)\boldsymbol{e}_r + \left(r\ddot{\varphi} + 2\,\dot{r}\dot{\varphi}\right)\boldsymbol{e}_\varphi$$

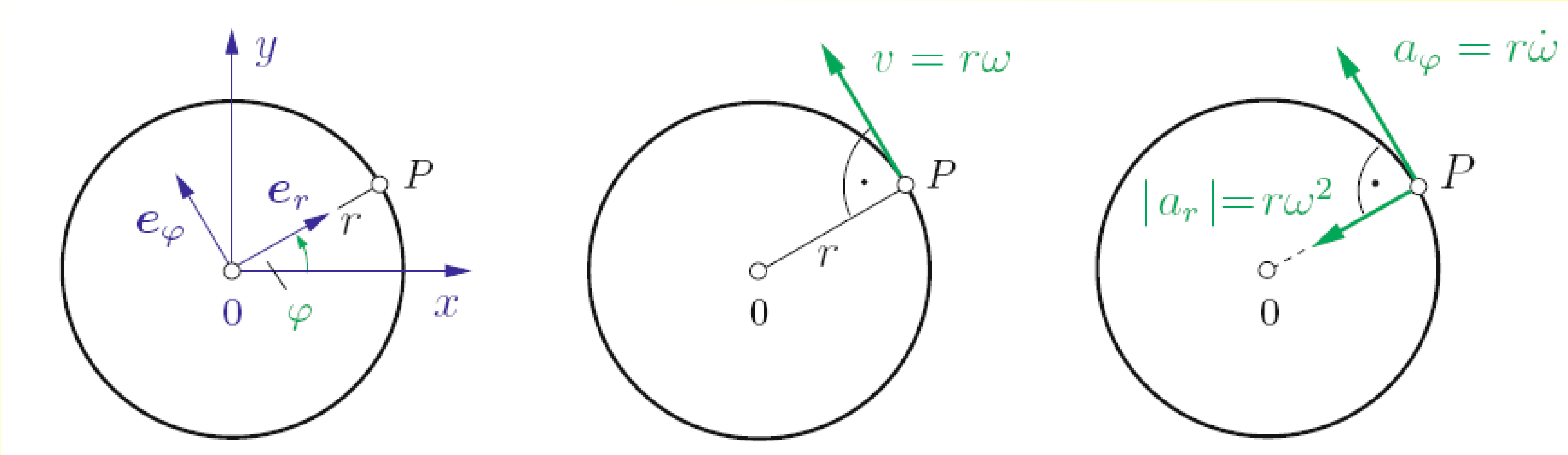
Nebenbetrachtung



$$\mathrm{d}\boldsymbol{e}_r = \mathrm{d}\varphi\,\boldsymbol{e}_\varphi \quad \rightarrow \quad \dot{\boldsymbol{e}}_r = \frac{\mathrm{d}\boldsymbol{e}_r}{\mathrm{d}t} = \frac{\mathrm{d}\varphi}{\mathrm{d}t}\,\boldsymbol{e}_\varphi = \dot{\varphi}\,\boldsymbol{e}_\varphi$$

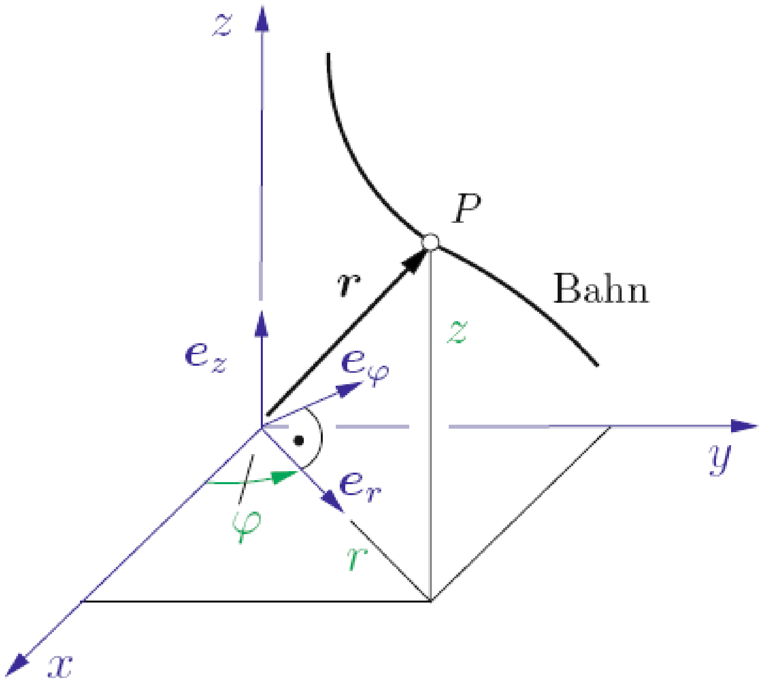
$$\mathrm{d}\boldsymbol{e}_\varphi = -\,\mathrm{d}\varphi\,\boldsymbol{e}_r \quad \rightarrow \quad \dot{\boldsymbol{e}}_\varphi = \frac{\mathrm{d}\boldsymbol{e}_\varphi}{\mathrm{d}t} = -\,\frac{\mathrm{d}\varphi}{\mathrm{d}t}\,\boldsymbol{e}_r = -\,\dot{\varphi}\,\boldsymbol{e}_r$$

Sonderfall: Kreisbewegung



$$\boldsymbol{r} = r\,\boldsymbol{e}_r, \quad \boldsymbol{v} = r\omega\,\boldsymbol{e}_\varphi, \quad \boldsymbol{a} = -r\omega^2\,\boldsymbol{e}_r + r\dot{\omega}\,\boldsymbol{e}_\varphi$$

2.1.4 Räumliche Bewegung



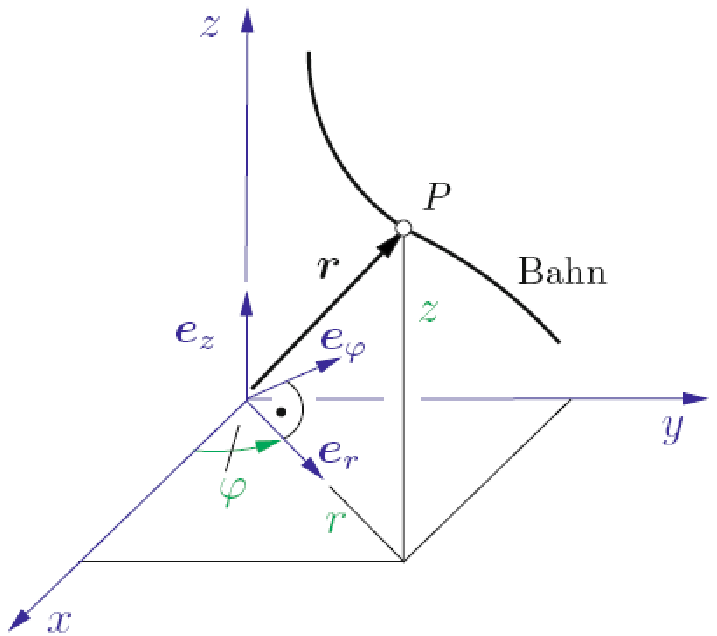
1.) Kartesisches Koordinatensystem

$$\boldsymbol{r}(t) = x(t)\,\boldsymbol{e}_x + y(t)\,\boldsymbol{e}_y + z(t)\,\boldsymbol{e}_z$$

$$\boldsymbol{v} = \dot{\boldsymbol{r}} = \dot{x}\,\boldsymbol{e}_x + \dot{y}\,\boldsymbol{e}_y + \dot{z}\,\boldsymbol{e}_z$$

$$\boldsymbol{a} = \dot{\boldsymbol{v}} = \ddot{\boldsymbol{r}} = \ddot{x}\,\boldsymbol{e}_x + \ddot{y}\,\boldsymbol{e}_y + \ddot{z}\,\boldsymbol{e}_z$$

Zylinderkoordinatensystem



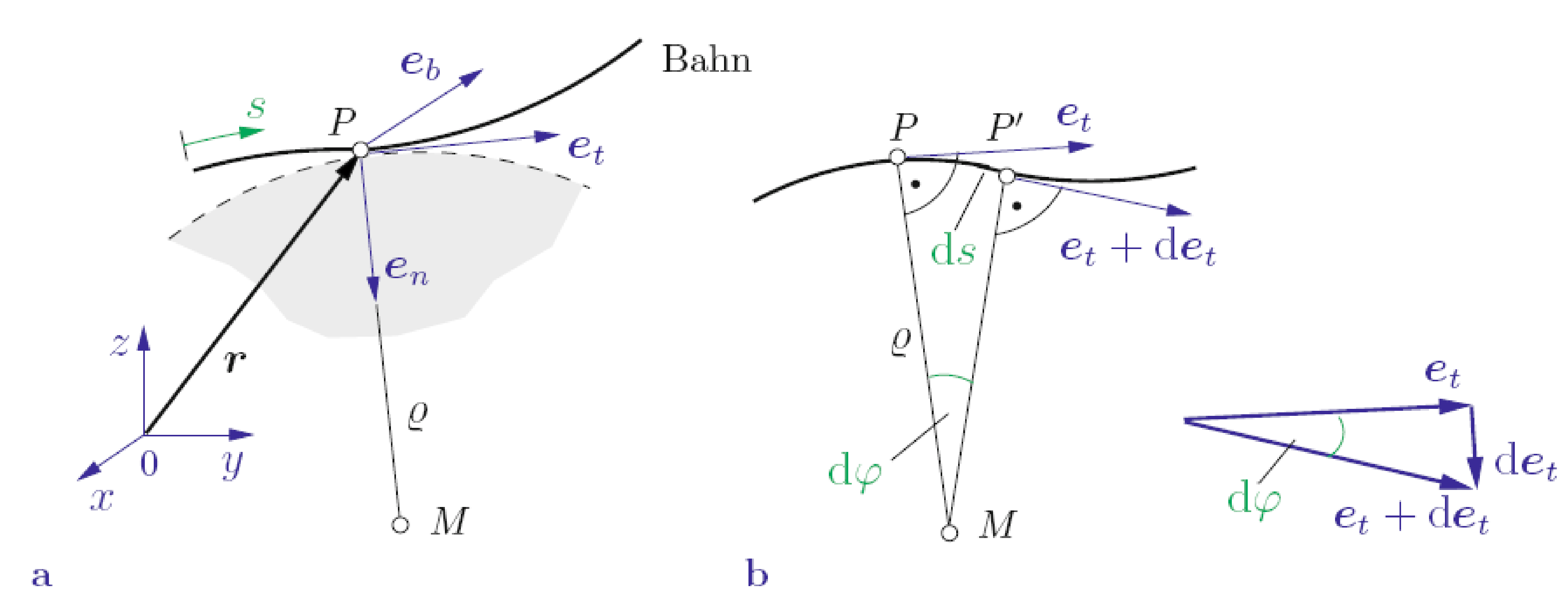
2.) Zylinderkoordinatensystem

$$\boldsymbol{r} = r\,\boldsymbol{e}_r + z\,\boldsymbol{e}_z\,,$$

$$\boldsymbol{v} = \dot{r}\,\boldsymbol{e}_r + r\dot{\varphi}\,\boldsymbol{e}_\varphi + \dot{z}\,\boldsymbol{e}_z\,,$$

$$\boldsymbol{a} = (\ddot{r} - r\dot{\varphi}^2)\,\boldsymbol{e}_r + (r\ddot{\varphi} + 2\,\dot{r}\dot{\varphi})\,\boldsymbol{e}_\varphi + \ddot{z}\,\boldsymbol{e}_z\,.$$

Natürliches Koordinatensystem



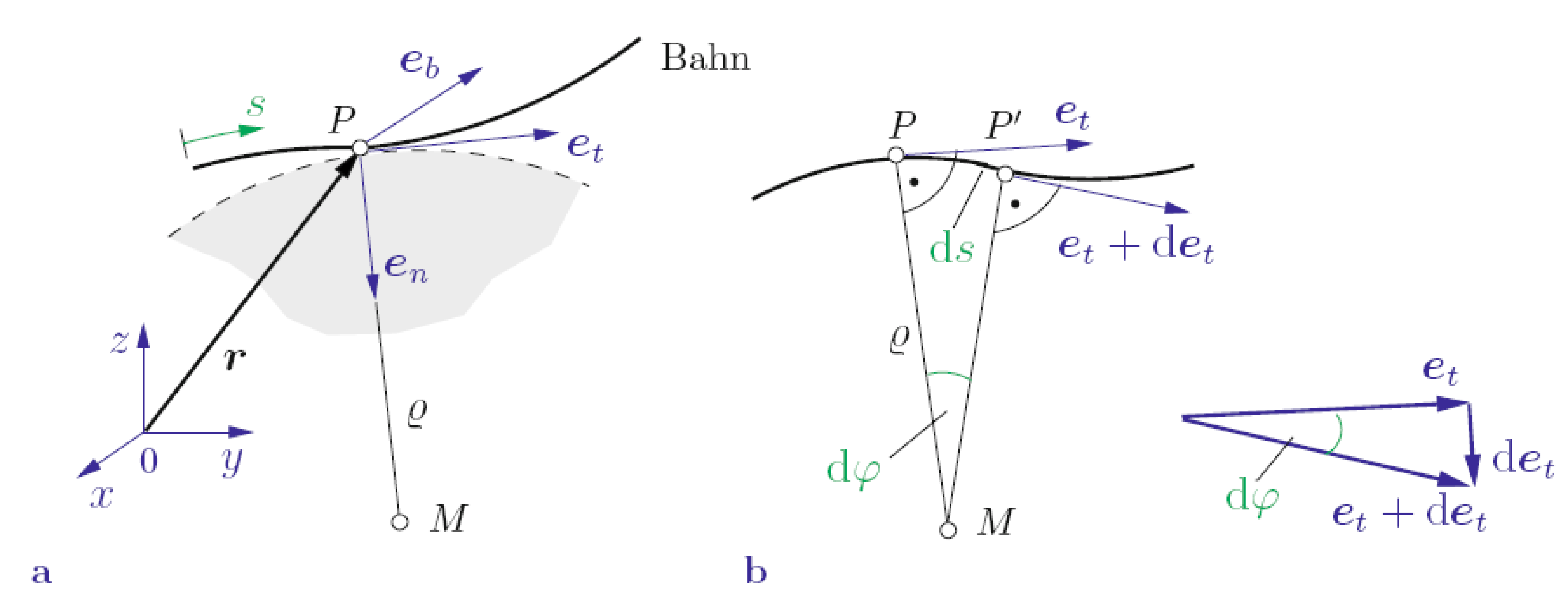
3.) Natürliches Koordinatensystem

$$\boldsymbol{r} = \boldsymbol{r}(s(t))$$

$$\boldsymbol{v} = v\,\boldsymbol{e}_t$$

$$\boldsymbol{a} = a_t\,\boldsymbol{e}_t + a_n\,\boldsymbol{e}_n = \dot{v}\,\boldsymbol{e}_t + \frac{v^2}{\rho}\,\boldsymbol{e}_n$$

Nebenbetrachtung



$$\mathrm{d}\boldsymbol{r} = \mathrm{d}s\,\boldsymbol{e}_t$$

$$\mathrm{d}\boldsymbol{e}_t = 1 \cdot \mathrm{d}\varphi\,\boldsymbol{e}_n = \frac{\mathrm{d}s}{\rho}\,\boldsymbol{e}_n \quad \longrightarrow \quad \dot{\boldsymbol{e}}_t = \frac{\mathrm{d}\boldsymbol{e}_t}{\mathrm{d}t} = \frac{1}{\rho}\frac{\mathrm{d}s}{\mathrm{d}t}\,\boldsymbol{e}_n = \frac{v}{\rho}\,\boldsymbol{e}_n$$

Analogie

Geradlinige Bewegung	*Räumliche Bewegung*
x	s
$v = \dot{x}$	$v = \dot{s}$
$a = \dot{v} = \ddot{x}$	$a_t = \dot{v} = \ddot{s}$

Daher können alle Formeln der geradlinigen Bewegung auf räumliche Bewegung in natürlichen Koordinaten angewendet werden!