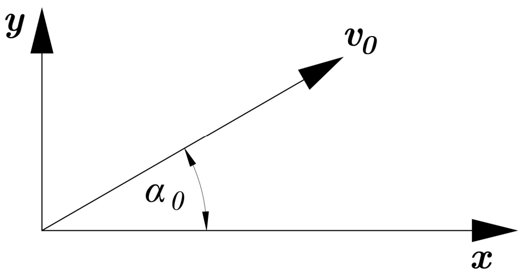
Aufg. 1

Schiefer Wurf:



$geg: v_0 = 200\,\frac{m}{s}$

$h_{AB} = 5\,m$

$t^* = 2{,}4237\,s$

$ges: l_{AB} = ?$

$\alpha_0 = ?$

$m \cdot \ddot{x} = 0 \quad \rightarrow \ddot{x} = 0$ (Beschleunigung)

$\dot{x}(t) = v_0 \cdot \cos\alpha_0 + 0$ (Geschwindigkeit)

$x(t) = v_0 \cdot \cos\alpha_0 \cdot t$ (Weg in x-Richtung)

$\not{m} \cdot \ddot{y} = -\not{m} \cdot g \quad \rightarrow \ddot{y} = -g$

$\dot{y}(t) = -g \cdot t + v_0 \cdot \sin\alpha_0$

$y(t) = -g \cdot \frac{t^2}{2} + v_0 \cdot \sin\alpha_0 \cdot t$

$$x(t = t^*) = v_0 \cdot \cos\alpha_0 \cdot t^* = l_{AB} \qquad (1)$$

$$y(t = t^*) = -g \cdot \frac{\left(t^*\right)^2}{2} + v_0 \cdot \sin\alpha_0 \cdot t^* = h_{AB} \qquad (2)$$

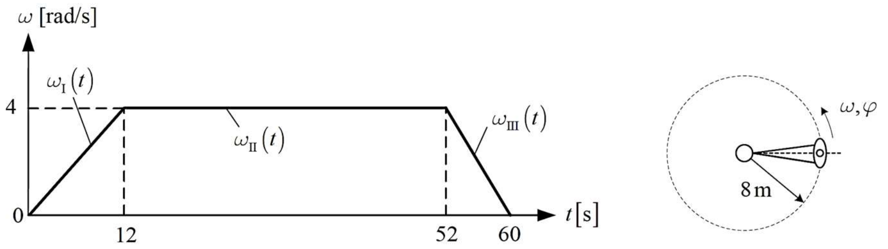
$$aus\,(2): -g \cdot \frac{2{,}4237^2}{2} + 200 \cdot \sin\alpha_0 \cdot 2{,}4237 = 5\,m \Leftrightarrow \alpha_0 = \arcsin\left(\frac{5 + g \cdot \frac{2{,}4237^2}{2}}{200 \cdot 2{,}4237}\right)$$

$$\underline{\underline{\alpha_0 = 4{,}0°}}$$

$$aus\,(1): 200 \cdot \cos(4{,}0°) \cdot 2{,}4237 = \underline{\underline{483{,}56\,m}}$$

Aufg. 2

geg:



a) Bereich I:

$\omega_I(t) = a \cdot t + b \quad \rightarrow \omega_I(t=0) = 0 \rightarrow b = 0 \quad$ (y-Achsabstand)

$$\rightarrow \omega_I(t=12) = a \cdot 12 = 4 \rightarrow a = \frac{4}{12} = \frac{1}{3} \quad \text{(Steigung)}$$

$$\Rightarrow \quad \omega_I(t) = \frac{1}{3} \cdot t$$

Bereich II:

$\omega_{II}(t) = 4 \quad (const.)$

Bereich III:

$\omega_{III}(t) = c \cdot t + d \quad \rightarrow \omega_{III}(t=0) = 4 \rightarrow d = 4 \quad$ (y-Achsabstand)

$$\rightarrow \omega_{III}(t=8) = 4 + c \cdot 8 = 0 \rightarrow c = -\frac{4}{8} = -\frac{1}{2} \quad \text{(Steigung)}$$

$$\Rightarrow \quad \omega_{III}(t) = 4 - \frac{1}{2} \cdot t$$

b) Allgemein : $\varphi(t) = \varphi_0 + \int_{t_0}^{t_1} \omega(t) dt$

Bereich I:

$$\varphi_I(t) = \varphi_0 + \int_0^t \omega_I(t) dt = 0 + \int_0^t \frac{1}{3} t^2 dt = 0 + \frac{1}{3} \cdot \frac{t^2}{2} = \frac{t^2}{6}$$

$$\varphi_I(t=12) = \frac{12^2}{6} = 24[rad]$$

Bereich II:

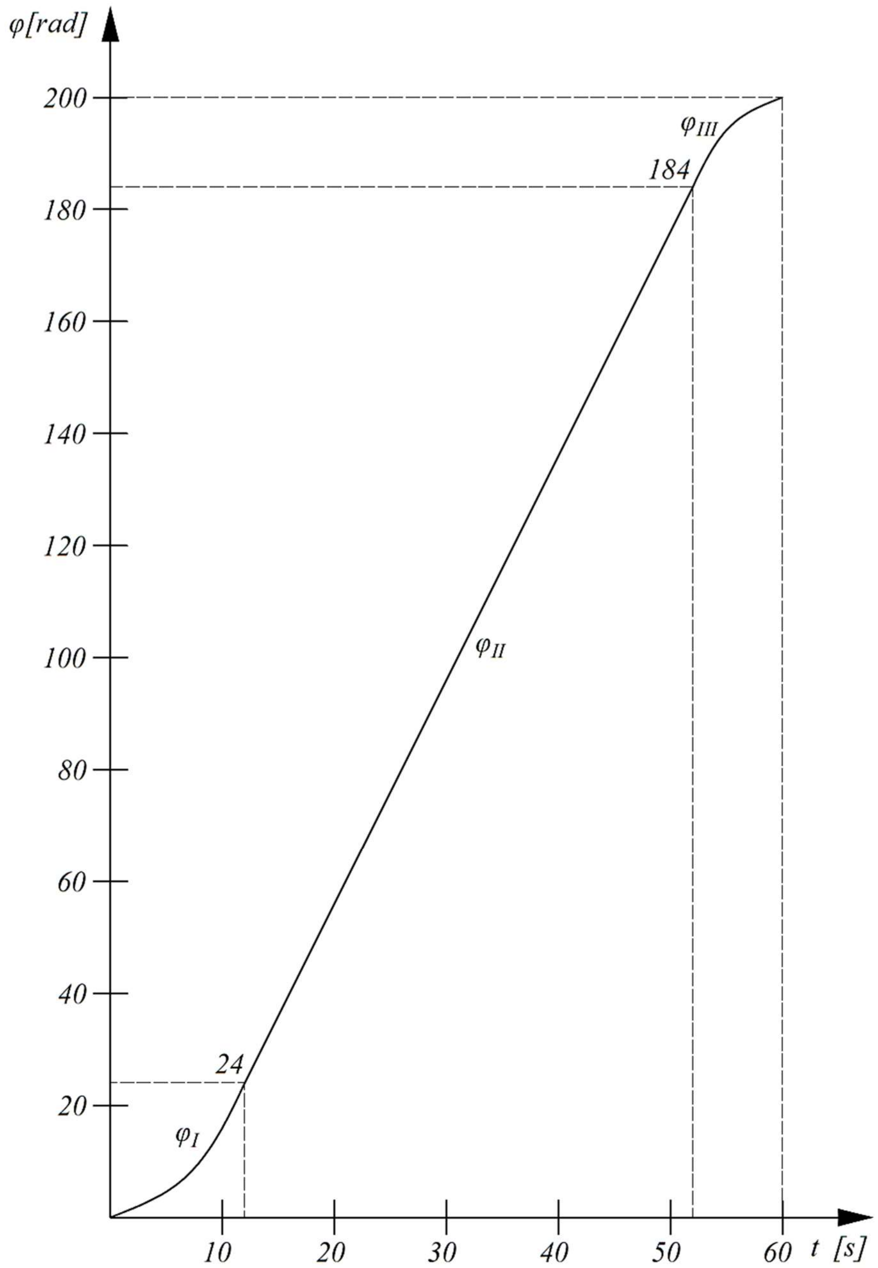
$$\varphi_{II}(t) = \varphi_0 + \int_0^t \omega_{II}(t) dt = 24 + \int_0^t 4 dt = 24 + 4 \cdot t$$

$$\varphi_{II}(t=40) = 24 + 4 \cdot 40 = 184[rad]$$

Bereich III:

$$\varphi_{III}(t) = \varphi_0 + \int_0^t \omega_{III}(t)dt = 184 + \int_0^t 4 - \frac{1}{2}t\,dt = 184 + 4t - \frac{1}{4}t^2$$

$$\varphi_{III}(t=8) = 184 + 4\cdot 8 - \frac{1}{4}\cdot 8^2 = 200\ [rad]$$



c) 1 Umdrehung $\hat{=} 360° \hat{=} 2\pi$

$$\rightarrow n = \frac{200}{2\pi} = 31{,}83\,\text{Umdrehungen}$$

d) $s = r\cdot\varphi(t) = 8\cdot 200 = 1600m$

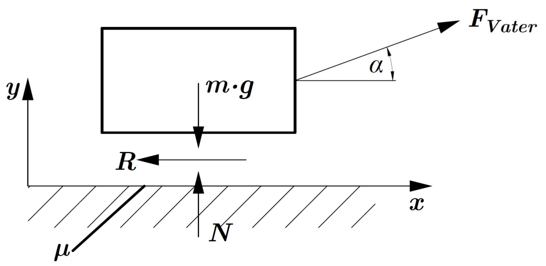
Aufg. 3

$$\left.\begin{aligned} m_{Schlitten} &= 5\,kg \\ m_{Kinder} &= 29\,kg \end{aligned}\right\} m_{ges} = 34\,kg \text{ (im Folgenden nur } m\text{)}$$

$$F_{Vater} = 40\,N$$

$$\sin\alpha = \frac{0,6}{2,0} \rightarrow \alpha = \arcsin\left(\frac{0,6}{2,0}\right) = 17,46°$$

a) Bewegungsgleichung:



$$\rightarrow: m \cdot \ddot{x} = F_{Vater} \cdot \cos(\alpha) - R$$

$$\uparrow: m \cdot \ddot{y} = m \cdot g - N - F_{Vater} \cdot \sin(\alpha) = 0$$

$$\Leftrightarrow N = m \cdot g - F_{Vater} \cdot \sin(\alpha)$$

$$= 34 \cdot 9,81 - 40 \cdot \sin(\alpha) = 321,54\,N$$

$$da\,R = \mu \cdot N\,gilt$$

$$\rightarrow m \cdot \ddot{x} = 40 \cdot \cos(\alpha) - \mu \cdot N = 40 \cdot \cos(\alpha) - 0,08 \cdot 321,54 = 12,434\,N$$

$$\Rightarrow \ddot{x} = \frac{12,434\,kg \cdot \frac{m}{s^2}}{34\,kg} = 0,366\,\frac{m}{s^2}$$

Energiesatz: $E_{k1} - E_{k0} = W \quad und\; W = \int_0^s \vec{F}\,ds$

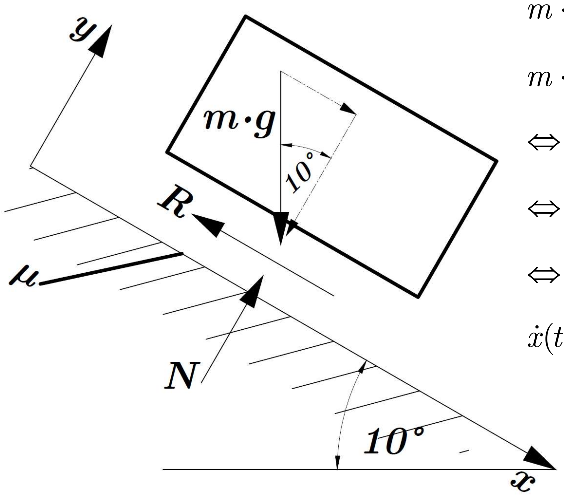
$$E_{k1} - \underbrace{E_{k0}}_{0} = \int_0^s \vec{F}\,ds$$

$$\Leftrightarrow \frac{m \cdot v^2}{2} = \int_0^{20} 12,434\,ds$$

$$\Leftrightarrow \frac{m \cdot v^2}{2} = 12,434 \cdot 20$$

$$\Leftrightarrow v = \sqrt{\frac{12,434 \cdot 20 \cdot 2}{34}} = \underline{\underline{3,825\,\frac{m}{s}}}$$

b)



$$m \cdot \ddot{y} = N - \cos 10° \cdot m \cdot g = 0 \rightarrow N = \cos 10° \cdot m \cdot g$$

$$m \cdot \ddot{x} = -R \cdot + \sin 10° \cdot m \cdot g \qquad da\, R = \mu \cdot N\, gilt$$

$$\Leftrightarrow m \cdot \ddot{x} = -\mu \cdot \cos 10° \cdot m \cdot g + \sin 10° \cdot m \cdot g$$

$$\Leftrightarrow \cancel{m} \cdot \ddot{x} = \cancel{m} \cdot g\left(-\mu \cdot \cos 10° + \sin 10°\right)$$

$$\Leftrightarrow \ddot{x} = 9,81 \cdot \left(-0,08 \cdot \cos 10° + \sin 10° = 0,094\right) = 0,9306$$

$$\dot{x}(t) = v(t) = 0,9306 \cdot t + 3,825$$

Aufg. 4

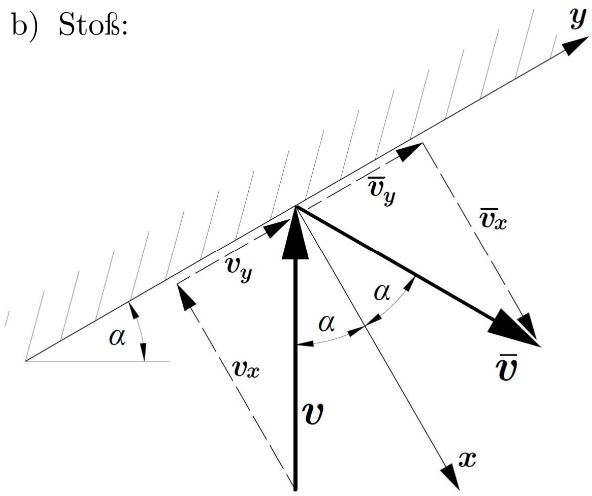
a) <u>*Energiesatz :*</u> $E_{k0} + E_{p0} = E_{k1} + E_{p1} = const.$

$$\rightarrow \underbrace{0}_{E_{k0}} + \underbrace{\frac{1}{2} c \cdot f^2}_{E_{p0}} = \underbrace{\frac{m \cdot v^2}{2}}_{E_{k1}} + \underbrace{m \cdot g \cdot h}_{E_{p1}}$$

$$\rightarrow \underbrace{0}_{E_{k0}} + \underbrace{\frac{1}{2} \cdot 100 \cdot 0,2^2}_{E_{p0}} = \underbrace{\frac{0,05 \cdot v^2}{2}}_{E_{k1}} + \underbrace{0,05 \cdot 9,81 \cdot (3,0 + 0,2)}_{E_{p1}}$$

$$\Leftrightarrow v = \sqrt{\frac{2 \cdot \left(\frac{1}{2} \cdot 100 \cdot 0,2^2 - 0,05 \cdot 9,81 \cdot 3,2\right)}{0,05}} = 4,149 \frac{m}{s}$$

b) Stoß:



$$v_x = -\cos(\alpha) = -4,149 \cdot \cos 30° = -3,593 \frac{m}{s}$$

$$v_y = -\sin(\alpha) = -4,149 \cdot \sin 30° = -2,075 \frac{m}{s}$$

$$\bar{v}_x = -e \cdot v_x = -0,75 \cdot (-3,593) = 2,695 \frac{m}{s}$$

$$\bar{v}_y = v_y = 2,075 \frac{m}{s}$$

$$\left|\bar{v}\right| = \sqrt{\bar{v}_x^{\,2} + \bar{v}_y^{\,2}} = \sqrt{2,695^2 + 2,075^2} = 3,401 \frac{m}{s}$$

$$\tan(\bar{\alpha}) = \frac{\bar{v}_y}{\bar{v}_x} \rightarrow \bar{\alpha} = \arctan\left(\frac{2,075}{2,695}\right) = \underline{\underline{37,6°}}$$

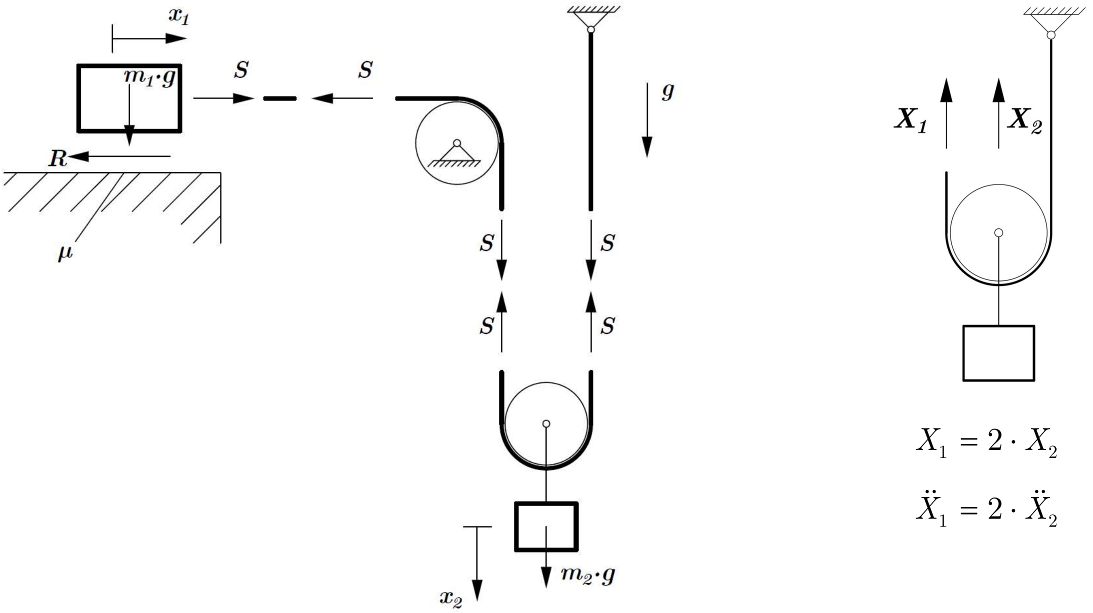
c) <u>*Energiesatz:*</u> $\underbrace{E_{k0}}_{\bar{v}} + \underbrace{E_{p0}}_{Höhe\,l} = \underbrace{E_{k1}}_{v_{Boden}} + \underbrace{E_{p1}}_{0} = const.$

$$\rightarrow \frac{1}{2} \cdot \not{m} \cdot \bar{v}^2 + \not{m} \cdot g \cdot l = \frac{1}{2} \cdot \not{m} \cdot v_{Boden}^2$$

$$\Leftrightarrow \frac{1}{2} \cdot 3{,}401^2 + 9{,}81 \cdot 4{,}0 = \frac{1}{2} \cdot v_{Boden}^2 \Leftrightarrow v_{Boden} = \sqrt{3{,}401^2 + 2 \cdot 9{,}81 \cdot 4{,}0} = \underline{\underline{9{,}489 \frac{m}{s}}}$$

Aufg. 5

a)



$$X_1 = 2 \cdot X_2$$

$$\ddot{X}_1 = 2 \cdot \ddot{X}_2$$

Die Seilkraft ist überall gleich!

$$m_1 \cdot \ddot{x}_1 = S - R = S - N \cdot \mu = S - m_1 \cdot g \cdot \mu$$

$$\rightarrow \ddot{x}_1 = \frac{S - m_1 \cdot g \cdot \mu}{m_1} = \frac{S}{6m} - 0{,}2 \cdot g$$

$$m_2 \cdot \ddot{x}_2 = m_2 \cdot g - 2 \cdot S$$

$$\rightarrow \ddot{x}_2 = g - \frac{2 \cdot S}{m_2} = g - \frac{2 \cdot S}{4m}$$

Bedingungsgleichung: $X_2 = \frac{X_1}{2}$; $\ddot{X}_2 = \frac{\ddot{X}_1}{2}$

$$\rightarrow \underbrace{g - \frac{2 \cdot S}{4m}}_{X_2} = \frac{1}{2} \cdot \underbrace{\left(\frac{S}{6m} - 0{,}2 \cdot g\right)}_{X_1}$$

$$\Leftrightarrow 2g - \frac{S}{m} = \frac{S}{6m} - 0{,}2 \cdot g$$

$$\Leftrightarrow 2 \cdot g + 0{,}2 \cdot g = \frac{S}{6m} + \frac{S}{m} = \frac{7S}{6m}$$

$$\Leftrightarrow S = \frac{2{,}2 \cdot g \cdot 6m}{7} = \underline{\underline{1{,}886 \cdot g \cdot m}}$$

$$\ddot{x}_1 = \frac{S}{6m} - 0,2 \cdot g$$

$$\rightarrow \ddot{x}_1 = 0,314 \cdot g - 0,2 \cdot g = 0,1143 \cdot g = 1,121 \frac{m}{s^2}$$

$$\ddot{x}_2 = g - \frac{S}{2m}$$

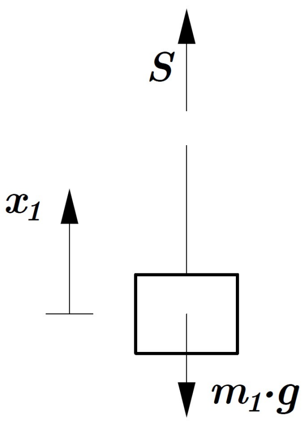
$$\rightarrow \ddot{x}_2 = g - \frac{1,886 \cdot g \cdot \cancel{m}}{2 \cdot \cancel{m}} = g - 0,943 \cdot g = 0,057 \cdot g = 0,559 \frac{m}{s^2}$$

b) $\dot{x}_1 = 1,121 \cdot t$

$$x_1 = \frac{1,121}{2} \cdot t^2 \Leftrightarrow \underbrace{2\,m}_{Strecke} = \frac{1,121}{2} \cdot t^2 \Leftrightarrow t = \sqrt{\frac{4}{1,121}} = \underline{\underline{1,889\,s}}$$

$$\dot{x}_1 = v(t) = 1,121 \cdot 1,889 = \underline{\underline{2,12 \frac{m}{s}}}$$

c)



$$m_1 \cdot \ddot{x}_1 = S - m_1 \cdot g$$

$$\Leftrightarrow \ddot{x}_1 = \frac{S}{m_1} - g = \frac{S}{6\,m} - g \qquad \ddot{x}_2 = g - \frac{S}{2 \cdot m} \text{(siehe oben)}$$

bei einer festen Rolle gilt : $X_1 = X_2$ bzw. $\ddot{X}_1 = \ddot{X}_2$

bei einer einseitig aufgehänten Rolle: $X_2 = \frac{X_1}{2}$; $\ddot{X}_2 = \frac{\ddot{X}_1}{2}$

$$\rightarrow \underbrace{g - \frac{S}{2 \cdot m}}_{X_2} = \frac{1}{2} \cdot \underbrace{\left(\frac{S}{6\,m} - g\right)}_{X_1} \Rightarrow 2 \cdot g - \frac{S}{m} = \frac{S}{6\,m} - g$$

$$\Leftrightarrow 3 \cdot g = \frac{S}{m} + \frac{S}{6 \cdot m} = \frac{7\,S}{6\,m}$$

$$\Rightarrow S = \frac{3 \cdot g \cdot 6 \cdot m}{7} = \frac{18}{7} \cdot g \cdot m = 2,571 \cdot g \cdot m$$

$$\ddot{x}_1 = \frac{S}{6\,m} - g \Leftrightarrow \frac{\frac{18}{7} \cdot g \cdot \cancel{m}}{6\,\cancel{m}} - g = -5,606 \frac{m}{s^2} \text{(Beschleunigung nach unten)}$$

$$\ddot{x}_2 = g - \frac{\frac{18}{7} \cdot g \cdot \cancel{m}}{2 \cdot \cancel{m}} = -2,803 \frac{m}{s^2} \text{(Beschleunigung nach oben)}$$