Baustatik I
Musterlösung Probeklausur 3

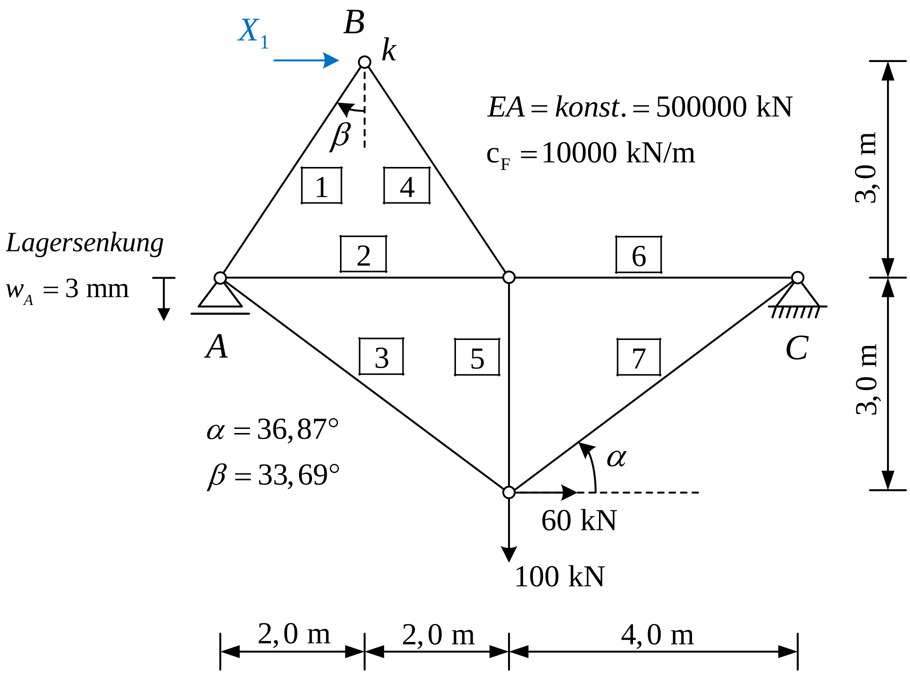
Aufgabe 1: Fachwerk

a) Grad der stat. Unbestimmtheit

$$a_a = 4 - 3 = 1$$
$$a_i = v - 3(n-1) = 18 - 3 \cdot (7-1) = 0$$

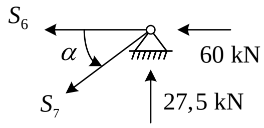
b) Stabkräfte und Auflagerreaktionen mithilfe des KGV

- Statisch bestimmtes Hauptsystem



- Nullzustand

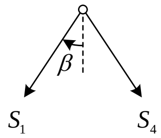
$$\curvearrowleft A: \quad -100 \cdot 4 + 60 \cdot 3 + C_V \cdot 8 = 0 \quad \Rightarrow \quad C_V = 27{,}5\,\mathrm{kN}$$
$$\uparrow: \quad A_V - 100 + 27{,}5 = 0 \quad \Rightarrow \quad A_V = 72{,}5\,\mathrm{kN}$$
$$\rightarrow: \quad C_H = 60\,\mathrm{kN}$$



$$\uparrow: \quad -S_7 \cdot \sin(\alpha) + 27,5 = 0 \qquad \Rightarrow \qquad S_7 = \quad 45{,}83\,\mathrm{kN}$$

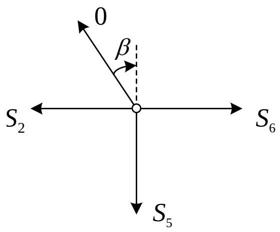
$$\rightarrow: \quad -S_6 - 60 - 45,83 \cdot \cos(\alpha) = 0$$

$$\Rightarrow \quad S_6 = -96{,}66\,\mathrm{kN}$$



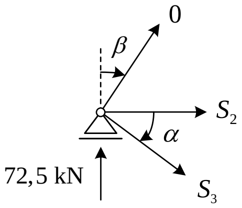
$$\uparrow: \quad S_1 = -S_4$$

$$\rightarrow: \quad S_1 = S_4 = 0\,\mathrm{kN}$$



$$\uparrow: \quad S_5 = 0$$

$$\rightarrow: \quad S_2 = S_6 = -96{,}66\,\mathrm{kN}$$



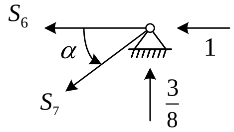
$$\uparrow: \quad 72,5 - \sin(\alpha) \cdot S_3 = 0 \qquad \Rightarrow \qquad S_3 = 120{,}83\,\mathrm{kN}$$

- Einheitszustand

$$\curvearrowleft A: \quad -1 \cdot 3 + C_V \cdot 8 = 0 \quad \Rightarrow \quad C_V = 3/8$$

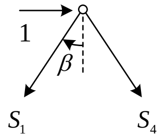
$$\uparrow: \quad A_V = -3/8$$

$$\rightarrow: \quad C_H = 1$$



$$\uparrow: \quad -S_7 \cdot \sin(\alpha) + 3/8 = 0 \qquad \Rightarrow \qquad S_7 = \quad 0,625$$

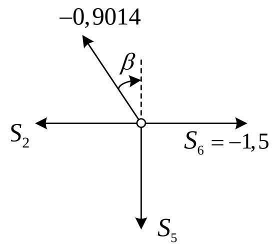
$$\rightarrow: \quad -S_6 - 1 - 0,625 \cdot \cos(\alpha) = 0 \quad \Rightarrow \quad S_6 = -1,5$$



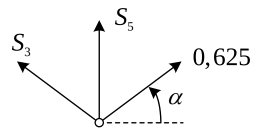
$$\uparrow: \quad -S_1 \cdot \cos(\beta) - S_4 \cdot \cos(\beta) = 0 \qquad \Rightarrow \qquad S_1 = -S_4$$

$$\rightarrow: \quad 1 - (-S_4) \cdot \sin(\beta) + S_4 \cdot \sin(\beta) = 0$$

$$\Rightarrow \quad S_4 = -0,9014 \quad S_1 = 0,9014$$



$$\uparrow: -S_5 + (-0,9014) \cdot \cos(\beta) = 0 \quad \Rightarrow \quad S_5 = -0,75$$
$$\rightarrow: \quad -S_2 + (-1,5) - (-0,9014)\sin(\beta) = 0$$
$$\Rightarrow \quad S_2 = -1,00$$



$$\rightarrow: -S_3 \cdot \cos(\alpha) + 0,625 \cdot \cos(\alpha) = 0$$
$$\Rightarrow \quad S_3 = 0,625$$

- Flexibilitätszahlen

$$\begin{aligned} EA_C\delta_{10} &= (-96,66)\cdot(-1)\cdot 4 + 120,83 \cdot 0,625 \cdot 5 + (-96,66)\cdot(-1,5)\cdot 4 \\ &\quad + 45,83 \cdot 0,625 \cdot 5 - 500000 \cdot \left(-\frac{3}{8}\right)\cdot(-0,003) \\ &= 924,9125 \end{aligned}$$

$$\begin{aligned} EA_C\delta_{11} &= 0,9014^2 \cdot \sqrt{13} + (-1)^2 \cdot 4 + 0,625 \cdot 5 + (-0,9014)^2 \cdot \sqrt{13} \\ &\quad + (-0,75)^2 \cdot 3 + (-1,5)^2 \cdot 4 + 0,625^2 \cdot 5 + \underbrace{\frac{EA_C}{c_F}}_{50} \cdot 1^2 \\ &= 74,453 \end{aligned}$$

- Bedingungsgleichung

$$X_1 = -\frac{924,9125}{74,453} = -12,423$$

- Rückrechnung

Stab	1	2	3	4	5	6	7
N_0 [kN]	0	-96,66	120,83	0	0	-96,66	45,83
N_1 [kN]	0,9014	-1,0	0,625	-0,9014	-0,75	-1,5	0,625
l [m]	$\sqrt{13}$	4	5	$\sqrt{13}$	3	4	5
N_{End} [kN]	-11,20	-84,24	113,07	11,20	9,32	-78,03	38,07

c) Horizontale Verschiebung des Knotens k

$$c_F = \frac{F}{w} \quad \Rightarrow \quad w = \frac{F}{c_F} = \frac{12,423}{10000}$$
$$= 1,2423 \cdot 10^{-3} \text{ m}$$

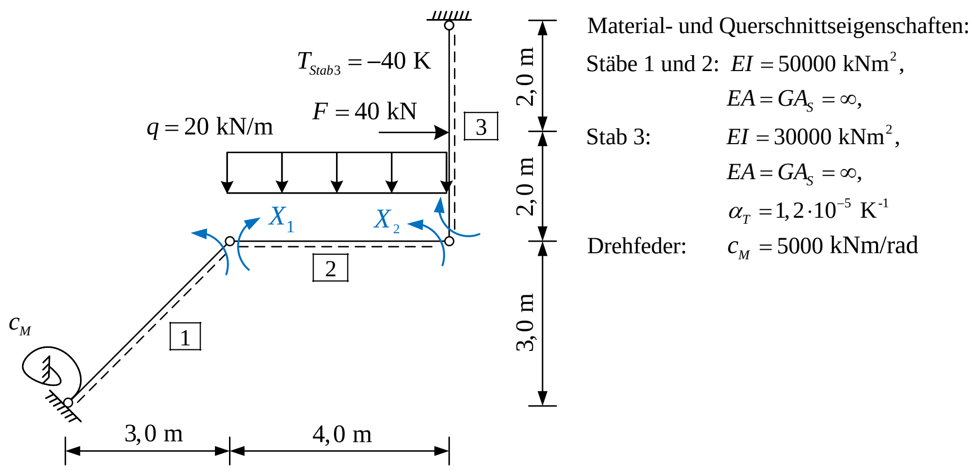
Aufgabe 2: Allgemeine Aufgabe zum Kraftgrößenverfahren

a) Biegemomentenverlauf mithilfe des KGV

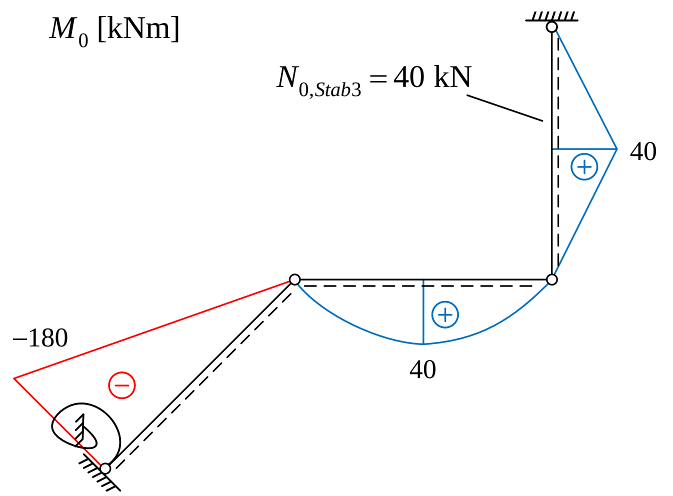
- Grad der stat. Unbestimmtheit

$$a = 5 - 3 = 2$$

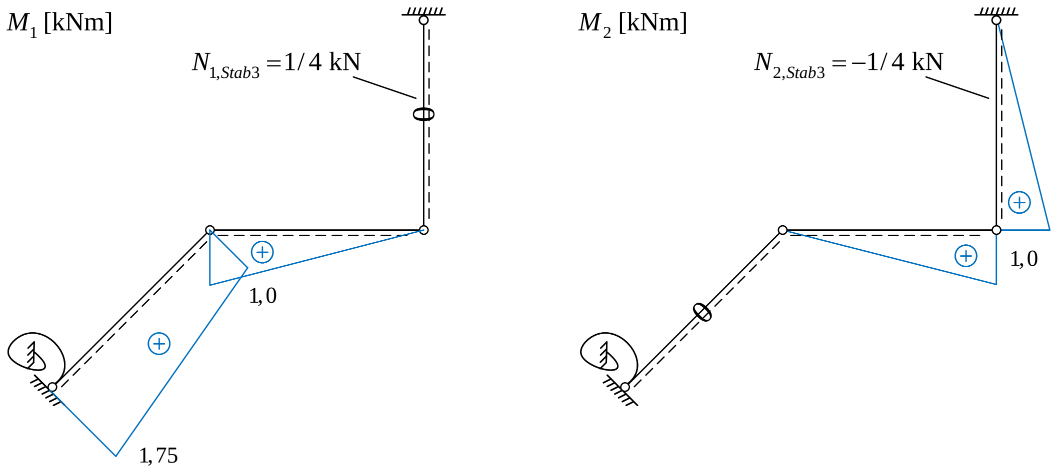
- Statisch bestimmtes Hauptsystem



- Nullzustand



- Einheitszustände



- Flexibilitätszahlen

$$EI_1 = EI_c = 50000, \quad \frac{EI_C}{EI_3} = \frac{5}{3}, \quad \frac{EI_C}{c_m} = 10$$

$$\begin{aligned} EI_C\delta_{10} &= \frac{1}{6} \cdot (-180) \cdot (2 \cdot 1,75 + 1) \cdot \sqrt{18} + \frac{1}{3} \cdot 40 \cdot 1 \cdot 4 \\ &\quad + 50000 \cdot \frac{1}{4} \cdot 1,2 \cdot 10^{-5} \cdot (-40) \cdot 4 - 180 \cdot 1,75 \cdot 10 \\ &= -3693,423 \end{aligned}$$

$$\begin{aligned} EI_C\delta_{20} &= \frac{1}{3} \cdot 40 \cdot 1 \cdot 4 + \frac{1}{6} \cdot 1 \cdot 40 \cdot (1 + 0,5) \cdot 4 \cdot \frac{5}{3} \\ &\quad + 50000 \cdot \left(-\frac{1}{4}\right) \cdot 1,2 \cdot 10^{-5} \cdot (-40) \cdot 4 \\ &= 144 \end{aligned}$$

$$\begin{aligned} EI_C\delta_{11} &= \frac{1}{3} \cdot (1,75^2 + 1,75 \cdot 1 + 1^2) \cdot \sqrt{18} + \frac{1}{3} \cdot 1^2 \cdot 4 + 1,75^2 \cdot 10 \\ &= 40,178 \end{aligned}$$

$$EI_C\delta_{22} = \frac{1}{3} \cdot 1^2 \cdot 4 + \frac{1}{3} \cdot 1^2 \cdot 4 \cdot \frac{5}{3} = 3,\overline{5} = \frac{32}{9}$$

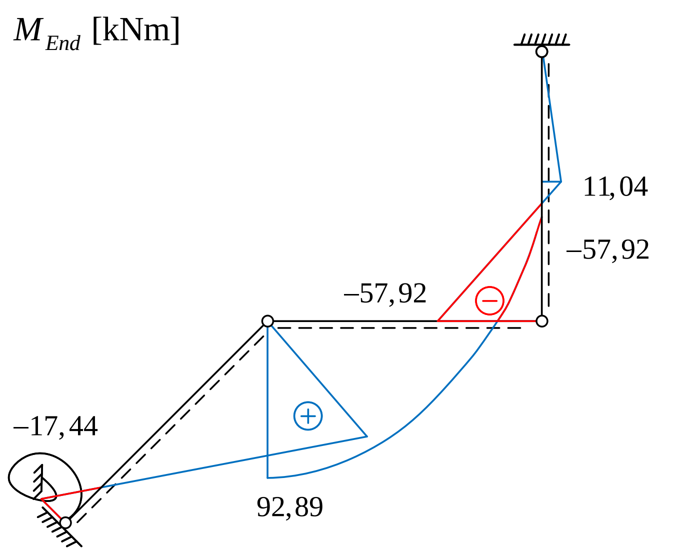
$$EI_C\delta_{21} = \frac{1}{6} \cdot 1^2 \cdot 4 = \frac{2}{3}$$

- Bedingungsgleichungen

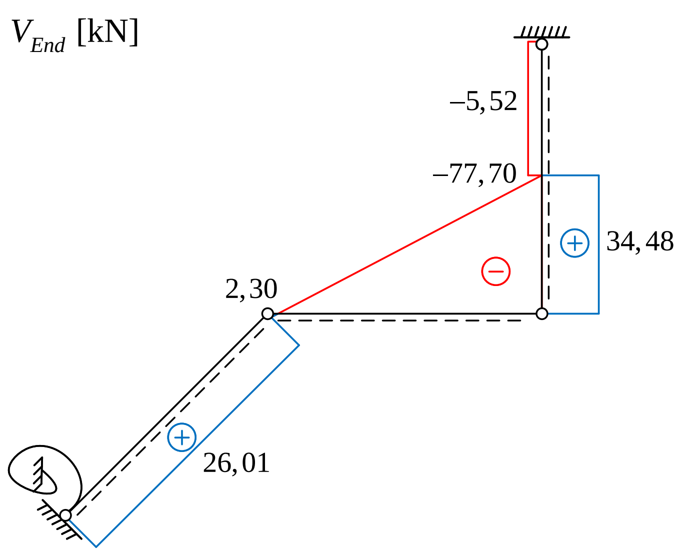
$$\begin{bmatrix} 40,178 & \frac{2}{3} \\ \frac{2}{3} & \frac{32}{9} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = - \begin{bmatrix} -3693,423 \\ 144 \end{bmatrix}$$

$$\Rightarrow \quad X_1 = 92,89 \qquad X_2 = -57,92$$

- Biegemomentenverlauf



b) Querkraftverlauf



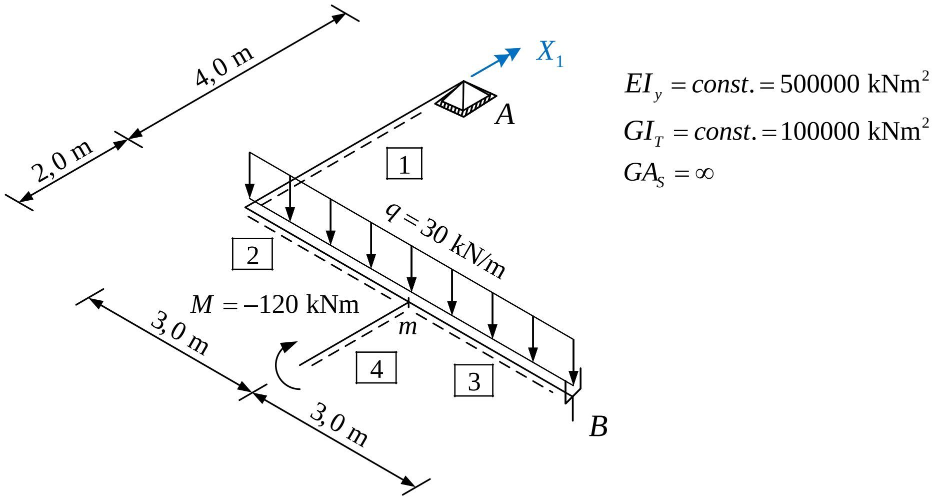
Aufgabe 3: Trägerrost

a) Biegemomenten- und Torsionsmomentenverlauf mithilfe des KGV

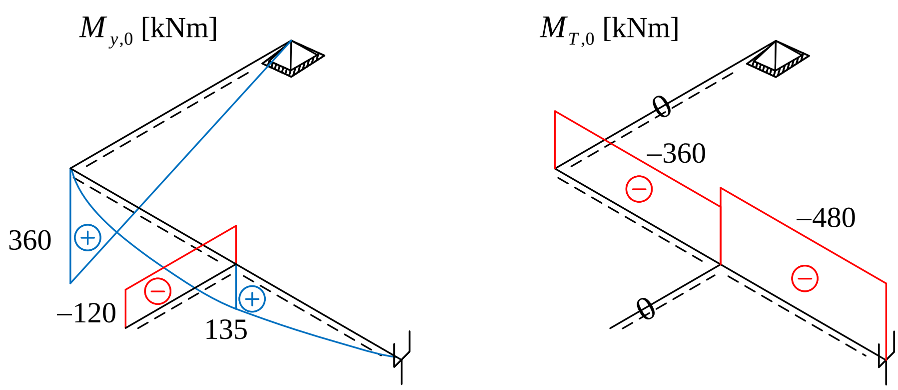
- Grad der stat. Unbestimmtheit

$$a = 4 - 3 = 1$$

- Statisch bestimmtes Hauptsystem



- Nullzustand

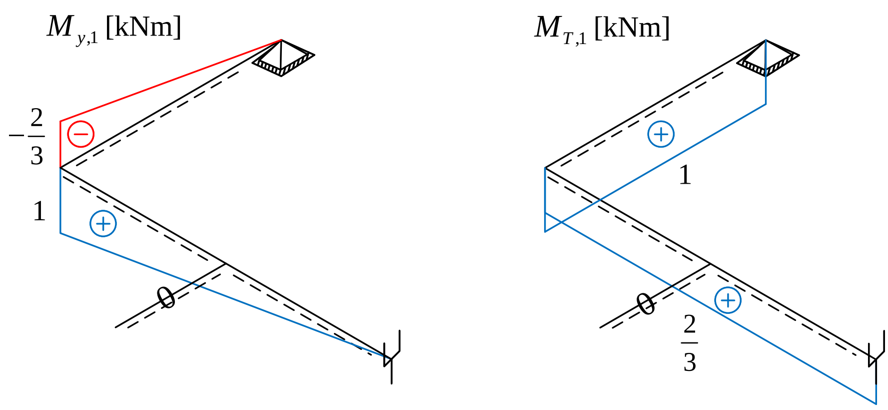


$$\overset{\curvearrowleft}{A_x}: \quad -30 \cdot 6 \cdot 3 + B_V \cdot 6 = 0 \qquad \Rightarrow \qquad B_V = 90\,\text{kN}$$

$$\uparrow: \quad A_V = 30 \cdot 6 - 90 = 90\,\text{kN}$$

$$\overset{\curvearrowleft}{B_y}: \quad A_v \cdot 4 - (-120) + M_{T,B} = 0 \qquad \Rightarrow \qquad M_{T,B} = -480\ \text{kNm}$$

- Einheitszustand



$$\overset{\curvearrowleft}{B_x}: \quad -A_v \cdot 6 - 1 = 0 \quad \Rightarrow \quad A_v = -1/6 \text{ kN} \quad \Rightarrow \quad B_v = 1/6 \text{ kN}$$

- Flexibilitätszahlen

$$\frac{EI_C}{GI_T} = 5$$

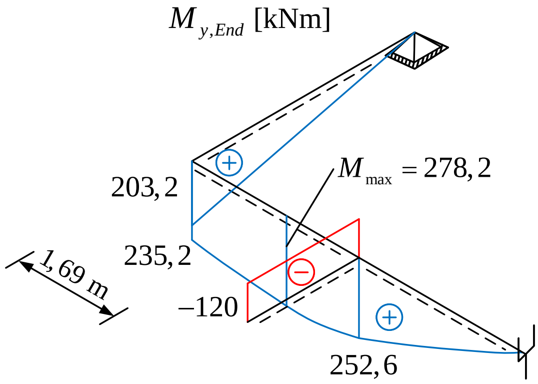
$$\begin{aligned} EI_C\delta_{10} &= \frac{1}{3} \cdot 360 \cdot \left(-\frac{2}{3}\right) \cdot 4 + \frac{1}{3} \cdot 135 \cdot 1 \cdot 6 \\ &\quad + (-360) \cdot \frac{2}{3} \cdot 3 \cdot 5 - 480 \cdot \frac{2}{3} \cdot 3 \cdot 5 \\ &= -8450 \end{aligned}$$

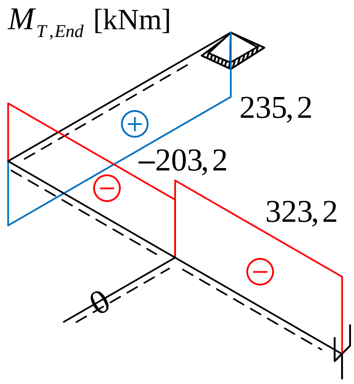
$$\begin{aligned} EI_C\delta_{11} &= \frac{1}{3} \cdot \left(-\frac{2}{3}\right)^2 \cdot 4 + \frac{1}{3} \cdot 1^2 \cdot 6 \\ &\quad + 1^2 \cdot 4 \cdot 5 + \left(\frac{2}{3}\right)^2 \cdot 6 \cdot 5 \\ &= 35,926 \end{aligned}$$

- Bedingungsgleichung

$$\Rightarrow \quad X_1 = -\frac{-8450}{35,926} = 235,206$$

- Rückrechnung



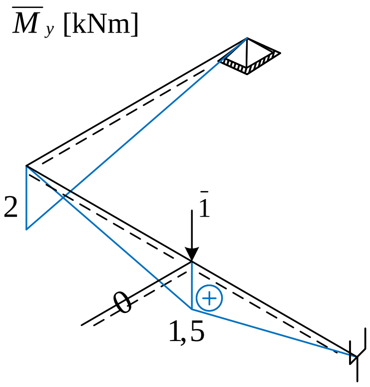


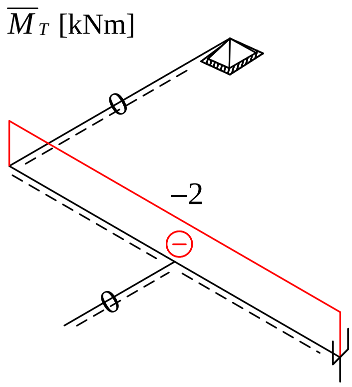
Berechnung von M_{max} :

$$V_i = \frac{-235,2}{6} + \frac{30 \cdot 6}{2} = 50,8 \text{ kN} \quad \Rightarrow x_{max} = \frac{50,8}{30} = 1,69 \text{ m}$$

$$M_{max} = 235,2 + 30 \cdot \frac{1,69^2}{2} = 278,2 \text{ kNm}$$

b) Durchbiegung w_m





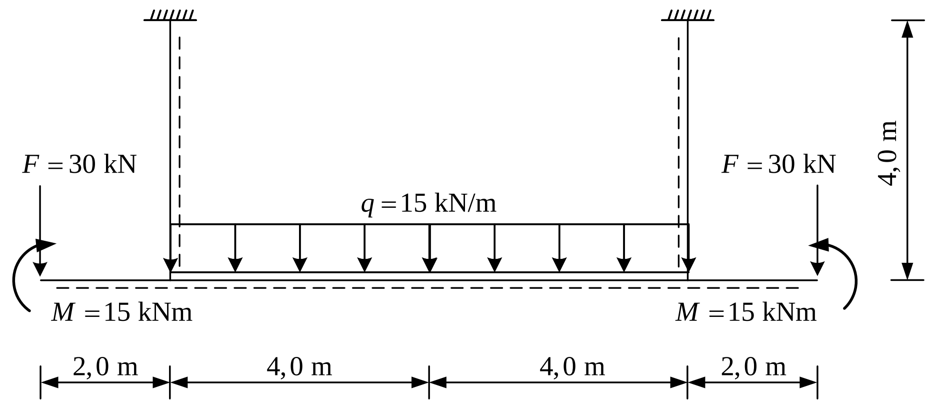
$$\begin{aligned} EIw_m = & \frac{1}{3} \cdot 203,2 \cdot 2 \cdot 4 + \frac{1}{6} \cdot \frac{3}{2} \cdot [235,2 \cdot 0,5 \\ & + 2 \cdot 252,6 - 0,25 \cdot (235,2 - 2 \cdot 252,6)] \cdot 6 \\ & + (-203,2) \cdot (-2) \cdot 3 \cdot 5 + (-323,2) \cdot (-2) \cdot 3 \cdot 5 \\ = & 17369,3 \end{aligned}$$

$$\Rightarrow \quad w_m = 3,47 \text{ cm}$$

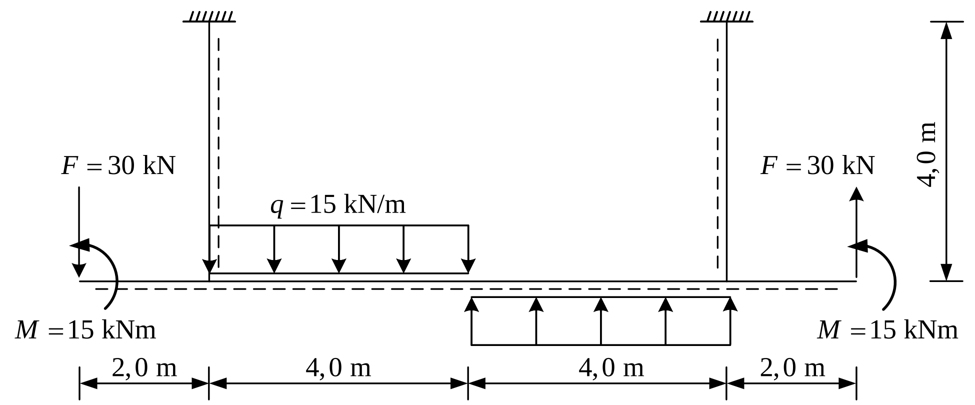
Aufgabe 4: Symmetrie und Antimetrie

a) Aufteilung in symmetrischen und antimetrischen Lastfall

Symmetrischer Lastfall:

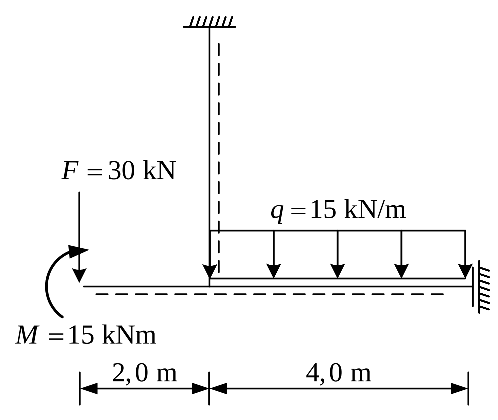


Antimetrischer Lastfall:

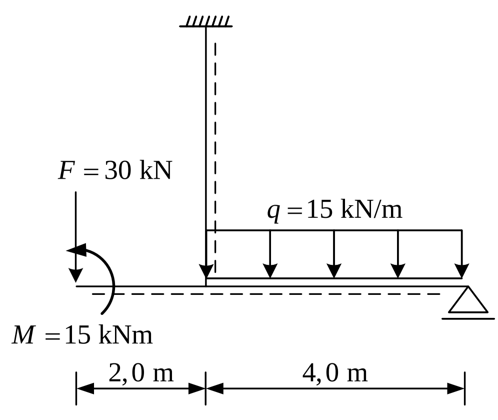


b) Ersatzsysteme

Ersatzsystem symmetrischer Lastfall:



Ersatzsystem antimetrischer Lastfall:

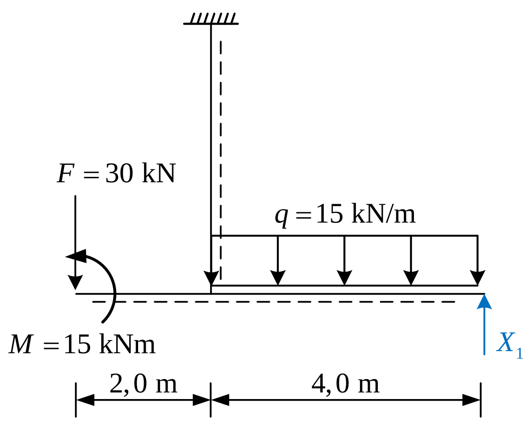


c) Grad der statischen Unbestimmtheit

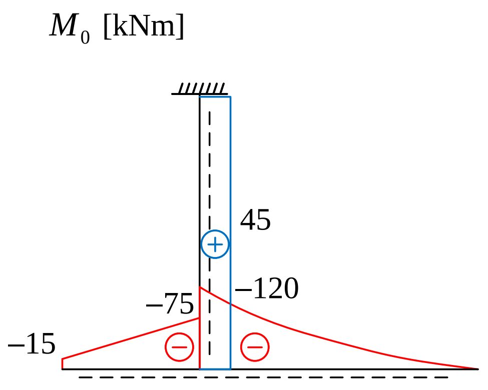
$$a_{ges} = 6 + 0 - 3 = 3$$
$$a_{sym} = 5 + 0 - 3 = 2$$
$$a_{anti} = 4 + 0 - 3 = 1$$

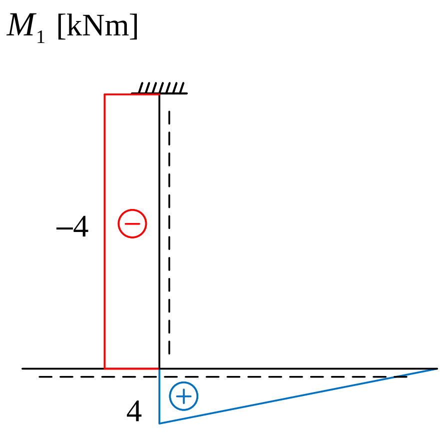
d) Schnittgrößenverläufe für den antimetrischen Lastfall

- Statisch bestimmtes Hauptsystem



- Null- und Einheitszustand



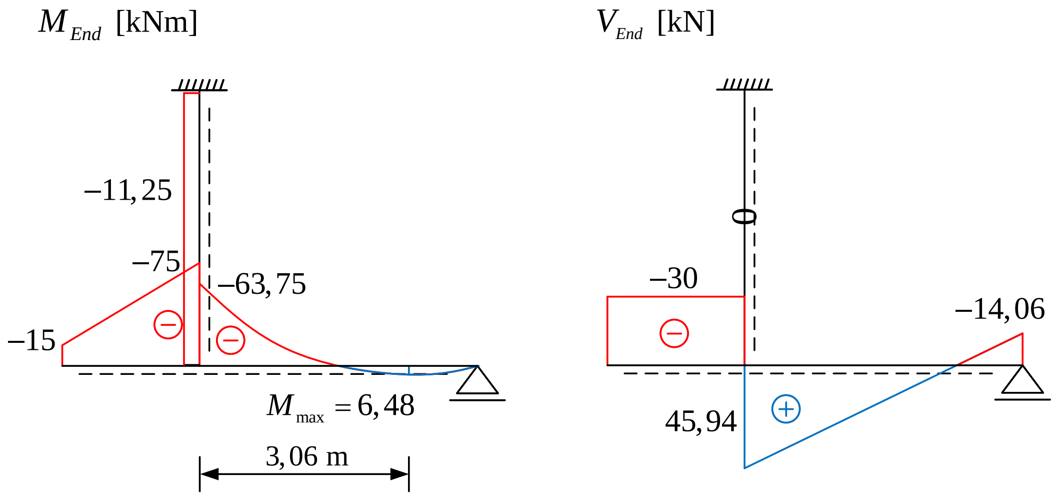


- Flexibilitätszahlen und Bedingungsgleichung

$$EI_C\delta_{10} = \frac{1}{4} \cdot (-120) \cdot 4 \cdot 4 + 45 \cdot (-4) \cdot 4 = -1200$$
$$EI_C\delta_{11} = \frac{1}{3} \cdot 4^2 \cdot 4 + (-4)^2 \cdot 4 = 85,\overline{3}$$

$$\Rightarrow \quad X_1 = -\frac{-1200}{85,\overline{3}} = 14,0625$$

- Rückrechnung



Berechnung von M_{max} :

$$V_i = \frac{63,75}{4} + \frac{15 \cdot 4}{2} = 45,94 \text{ kN} \quad \Rightarrow x_{max} = \frac{45,94}{15} = 3,06 \text{ m}$$

$$M_{max} = -63,75 + 15 \cdot \frac{3,06^2}{2} = 6,48 \text{ kNm}$$