Statistical Evaluation of UHPFRC Shear Verification Methods

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A database containing shear tests on UHPFRC beams is statistically evaluated using different verification methods, all superimposing the shear resistances provided by the concrete member, the yielding stirrups, and the fibers activated in the shear crack. Namely, the verification method which is expected to be considered with the forthcoming German DAfStb Guideline on Ultra-High Performance Concrete as well as the verification methods acc. to NF P18-710 and ÖBV Guideline "UHPC" are compared for beams with I-shaped and approximately rectangular solid cross-section. The analysis shows that the proposed method predicts the experimental shear resistances with sufficient reliability and less variation than the verification methods acc. to NF P18-710 and ÖBV Guideline "UHPC".

Keywords: beam, database, design, fiber, shear, UHPC, UHPFRC

1 Introduction

For Ultra-High Performance Fiber Reinforced Concrete (UHPFRC) beams several methods for verifying the shear resistance are available, e. g. [1-3]. These methods have in common to superimpose the shear resistances provided by aggregate interlock or shear friction, resistance of the compression zone, dowel action of the longitudinal reinforcement, longitudinal stress, yielding of the shear reinforcement (stirrups), and contribution of the fibers, however, in different ways. Due to the complexity of the interaction of the different shear resistance mechanisms, a generally accepted verification method for UHPFRC does not exist so far. Thus, standardization of a verification method requires careful database-driven validation in order to ensure sufficient reliability of the predicted shear resistance [1,4,5].

2 Database for UHPC and UHPFRC beams failing in shear

In [1] a shear database is presented which includes 31 studies with 208 tests on UHPC or UHPFRC beams failing in shear (Table 1). With regard to evaluation, distinction is made between tests on beams with I-shaped cross-section and tests on beams with compact cross-section (approximately rectangular solid cross-section). Acquisition, interpretation and normalization of the data are thoroughly discussed in [1].

Table 1: Configuration of tests included in the UHPC and UHPFRC shear database [1].

Configuration	I-shaped cross-section	Compact cross-section	Total
Total number of tests	133	75	208
Tests with/without fibers	114/19	56/19	170/38
Tests with/without prestressing	70/63	9/66	79/129
Tests with/without stirrups	28/105	11/64	39/169

The UHPFRC mixtures contain smooth straight steel fibers with a fiber length $6 \le l_f \le 40$ mm, a fiber diameter $0.15 \le \phi_f \le 0.50$ mm, and a fiber volume fraction $0.4 \le \rho_f \le 3.0$ %. The mean compressive cylinder strength of UHPC and UHPFRC ranges between f_{cm} = 118 and 224 MPa and the basic value of the post-cracking tensile strength of UHPFRC ranges between f_{cft0} = 1.3 and 16 MPa. Both strength values were converted from the data reported in the studies taking into account the type of test and the size of the specimen [1].

The shear span-to-depth ratio a_v/d of the beams is between 1.5 and 5.4.

3 Shear verification method proposed for the German DAfStb Guideline

The verification method proposed for the German DAfStb Guideline (*"proposed method"*) is presented in detail in [1,5]. It follows the abovementioned idea of superimposing different design shear resistances:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} + V_{Rd,f} \tag{1}$$

$V_{Rd,c}$ is the shear resistance of a member without shear reinforcement, defined similar to [6]:

$$V_{Rd,c} = \left[0.15/\gamma_C \cdot k \cdot \left(100 \cdot \rho_l \cdot f_{ck}[\text{MPa}]\right)^{1/3} + 0.12 \cdot \sigma_{cp}[\text{MPa}]\right] \cdot b_w \cdot d \tag{2}$$

$$k = 1 + \sqrt{200/d[\text{mm}]} \leq 2.0 \tag{3}$$

$$\rho_l = A_s / \left(b_w \cdot d\right) \leq 0.06 \tag{4}$$

$$\sigma_{cp} = N_{Ed}/A_c < 0.2 f_{cd}; \quad N_{Ed} > 0 \text{ for compression} \tag{5}$$

where f_{ck} and f_{cd} are the characteristic and the design value of the compressive cylinder strength, b_w is the smallest width of the cross-section in the tensile area, d is the effective depth of cross-section, γ_C is the partial factor for concrete, A_s and A_c are the cross-sectional areas of longitudinal reinforcement and concrete, and N_{Ed} is the axial force due to loading or prestressing.

The shear resistance provided by the shear reinforcement (stirrups) $V_{Rd,s}$ follows [6]:

$$V_{Rd,s} = A_{sw}/s \cdot z \cdot f_{ywd} \cdot \left(\cot\theta + \cot\alpha\right) \cdot \sin\alpha \tag{6}$$

$$z = 0.9d \leq \max\left\{d - 2c_{V,l}; d - c_{V,l} - 30\text{ mm}\right\} \tag{7}$$

$$\cot\theta = 1.2 + 2.4 \cdot \sigma_{cp} / f_{cd} \geq 1.0 \tag{8}$$

where A_{sw}, s, and f_{ywd} are the cross-sectional area, spacing, and design yield strength of the stirrups, α is the angle between the stirrups and the beam axis perpendicular to the shear force, and $c_{V,l}$ is the concrete cover of the longitudinal reinforcement in the concrete compression zone.

The shear resistance provided by the contribution of the fibers $V_{Rd,f}$ follows [7]:

$$V_{Rd,f} = b_w \cdot h \cdot \eta_F \cdot f_{cftd} \tag{9}$$

$$f_{cftd} = \alpha_{CF} \cdot f_{cftk}/\gamma_{CF} \tag{10}$$

$$f_{cftk} = \kappa_F \cdot f_{cft0} \tag{11}$$

where h is the height of the cross-section, η_F is a coefficient considering the shape of the shear crack, α_{CF} is the coefficient taking account of long-term effects on the post-cracking tensile strength, γ_{CF} is the partial factor for the post-cracking tensile strength, κ_F is a coefficient accounting for the fiber orientation, and f_{cft0} is the basic value of the post-cracking tensile strength.

Regarding the contribution of fibers, distinction is made between members with I-shaped and with compact cross-section. With I-shaped cross-section, multiple linear and parallel shear cracks develop in the thin web until further crack opening localizes into a single critical shear crack causing shear-tension failure. Prior to failure the contribution of fibers is almost uniform over the length of the critical shear crack, which is accounted for by $\eta_F = 1.0$. In contrast, UHPFRC beams with compact cross-section show an inclined shear crack which develops from a flexural crack initiated at the extreme tension fiber of the cross-section causing diagonal tension failure. The width of the critical shear crack increases from the crack tip, where the fibers may still be fully activated, towards the extreme tension fiber, where the fibers may already be partially pulled-out. This variable contribution of the fibers is considered by $\eta_F = 0.7$.

The fiber orientation is influenced by the casting method and the formwork geometry. For beams with thin web, the fiber orientation can be expected in the plane of the web, which is favorable for shear transfer. In contrast, a more isotropic fiber orientation may be expected for beams with wide web or compact cross-section. Therefore, κ_F = 1.0 is assumed for beams with I-shaped cross-section and κ_F = 0.5 (following [7]) for beams with compact cross-section.

4 Statistical evaluation of shear verification methods

158 data sets of the abovementioned database represent tests with fibers (UHPFRC beams) and $a_v/d \geq 2$. These data sets are selected for examining the *proposed method* and the methods acc. to *NF P18-710* [2] ("*NFP*") and *ÖBV Guideline "UHPC"* [3] ("*ÖBV*"). Evaluation of all data sets by means of the *proposed method* is presented in [1].

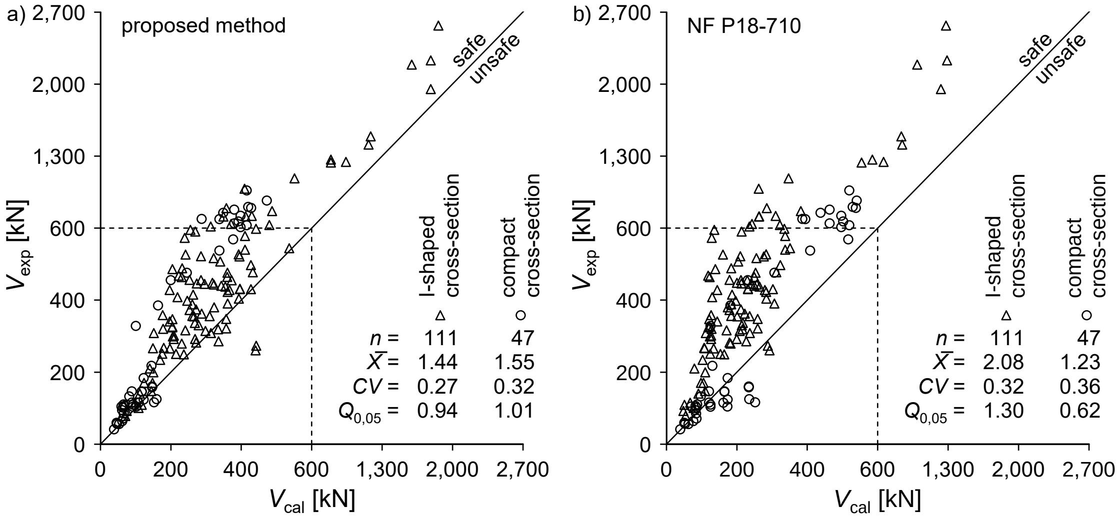
For comparison with test data, the following adjustments are applied: Mean values of material strength are used instead of characteristic or design values. For the *proposed method*, the factor 2.4 in Eq. (8) is replaced by 4.0 ($\approx 2.4\ f_{cm}/f_{cd}$) and partial factors $\gamma_C = \gamma_S = \gamma_{CF} = 1.0$ and $\alpha_{CF} = 1.0$ are applied. For *NFP*, the term $1/w^* \int_0^{w^*} \sigma_f(w)\,dw$ (where $w^* = 0.3$ mm) is approximated with $0.935 f_{cft0}$ which is a mean value found by integrating typical stress-crack opening relations from direct tensile tests [8]. K is equated with 1.25 and partial factors $\gamma_{cf} = \gamma_E = \gamma_S = 1.0$ are applied. For *ÖBV*, $f_{cfk,nom}$ is equated with f_{cft0} and the fiber orientation coefficient η_i is equated with η_β acc. to Eq. (7-4) in [3] (with $\beta = 90° - \vartheta$ and η_1, η_2 acc. to Tab. 7-2 in [3]). Partial factors $\gamma_c = \gamma_s = \gamma_{cf} = \gamma_{local} = 1.0$ and γ_η =1.2 are applied.

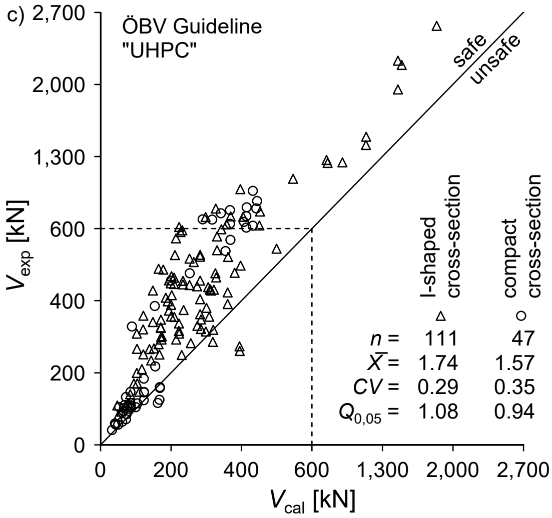
For each test, the shear resistance from experiment V_{exp} is compared with the theoretical shear resistance V_{cal} predicted by the three verification methods (Fig 1). The triangles and circles represent tests on beams with I-shaped and compact cross-section, respectively. For each verification method, the mean value $\bar{X}$, the coefficient of variation *CV*, and the 5 % quantile $Q_{0,05}$ of V_{exp}/V_{cal} is evaluated separately for both types of cross-section as well as for all 158 data sets.

For a total of 111 beams with I-shaped cross-section, the mean value of V_{exp}/V_{cal} is between $\bar{X}$ = 1.44 (*proposed method*) and 2.08 (*NFP*), *CV* is between 0.27 (*proposed method*) and 0.32 (*NFP*), and $Q_{0,05}$ is between 0.94 (*proposed method*) and 1.30 (*NFP*). A total of 47 beams with compact cross-section shows $\bar{X}$ between 1.23 (*NFP*) and 1.57 (*ÖBV*), *CV* between 0.32 (*proposed method*) and 0.36 (*NFP*), and $Q_{0,05}$ between 0.62 (*NFP*) and 1.01 (*proposed method*). The *proposed method* shows the smallest *CV* for both the beams with I-shaped cross-section and the beams with compact cross-section. Values $Q_{0,05}$ above or close to 1.0 reveal that both the *proposed method* and *ÖBV* predict the experimental shear resistances with sufficient reliability. In contrast, $Q_{0,05}$ is far below 1.0 when evaluating the beams with compact cross-section acc. to *NFP*. When considering all 158 data sets, the *proposed method* again shows the smallest coefficient of variation while *NFP* provides the largest scatter and the smallest reliability.

5 Summary and conclusions

The shear verification method proposed for the German DAfStb Guideline and the verification methods acc. to NF P18-710 and ÖBV Guideline "UHPC" are evaluated by means of 158 data sets of shear tests on UHPFRC beams. Comparison of the statistical parameters reveal that the *proposed method* predicts the experimental shear resistances with less variation than *NFP* and *ÖBV* for both the beams with I-shaped and approximately rectangular solid cross-section. *NFP* provides highly conservative results for beams with I-shaped cross-section, while the 5 % quantile of V_{exp}/V_{cal} is far below 1.0 for the beams with compact cross-section. *ÖBV* shows similar 5 % quantiles as the *proposed method* for both types of cross-section. However, the mean value of V_{exp}/V_{cal} is closer to 1.0 for the *proposed method* than for *ÖBV*. This means that the *proposed* *method* offers higher accuracy. Based on the statistical parameters it is concluded that the *proposed method* is sufficiently reliable for being considered with the forthcoming German DAfStb Guideline on Ultra-High Performance Concrete.





Statistical evaluation of V_{exp}/V_{cal}

	proposed method	NF P18-710	ÖBV-Guideline „UHPC“
n =	158	158	158
$\bar{X}$ =	1.47	1.83	1.69
CV =	0.29	0.40	0.31
$Q_{0,05}$ =	0.95	0.83	0.99

Figure 1: Evaluation of V_{exp}/V_{cal} with a) the proposed method [1], b) NF P18-710 [2], and c) ÖBV Guideline “UHPC” [3] for 158 UHPFRC beams with $a_v/d \geq 2$ and I-shaped or compact cross-section.

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